CHAPTER 10: CIRCLES

Introduction

A circle is one of the most symmetric and widely used shapes in geometry. It appears in everyday life such as wheels, coins, clocks, and many engineering designs.

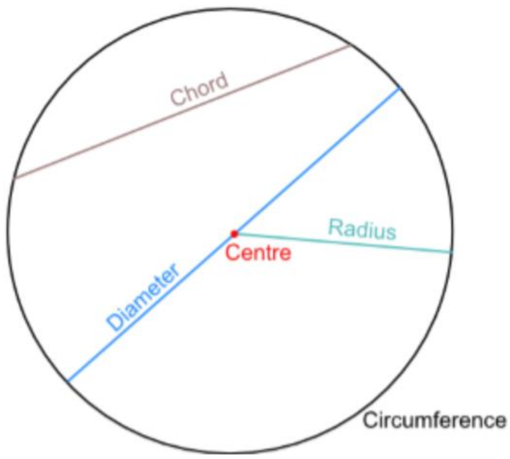
In this chapter, we study important properties of circles, especially **tangents**, their properties, and key theorems related to them.

Basic Definitions

Circle

A **circle** is a **closed plane figure** consisting of all points that are at a constant distance from a fixed point.

- Fixed point → **Centre (O)**
- Constant distance → **Radius (r)**



Important Terms

- **Radius:** Distance from centre to any point on circle
- **Diameter:** Twice the radius (longest chord)
- **Chord:** Line joining any two points on the circle
- **Arc:** A part of the circle
- **Segment:** Region between chord and arc

Circle and Line Positions

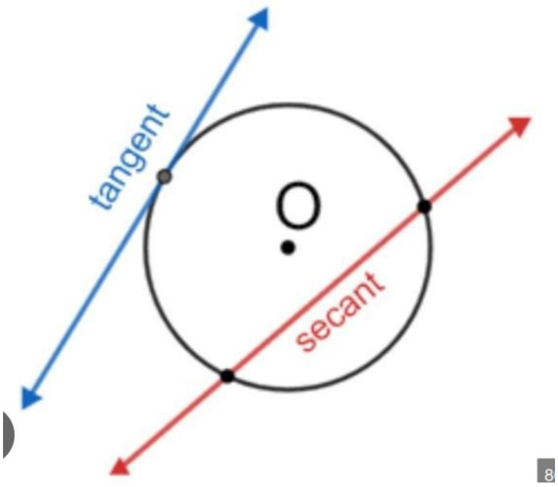
A line and a circle can have three possible positions:

1. **No intersection** → Line lies outside the circle
2. **One point intersection** → Line touches the circle (Tangent)
3. **Two points intersection** → Line cuts the circle (Secant)

Tangent to a Circle

A **tangent** is a line that touches the circle at **exactly one point**.

- That point is called **Point of Contact**
- Only **one tangent** can be drawn at a point on the circle



Secant

A **secant** is a line that intersects the circle at **two points**.

A tangent is actually a **special case of a secant** where the two points coincide.

Number of Tangents from a Point

1. Point inside the circle

- No tangent possible

2. Point on the circle

- Only **one tangent**

3. Point outside the circle

- Exactly **two tangents**

Theorem 1: Tangent ⊥ Radius

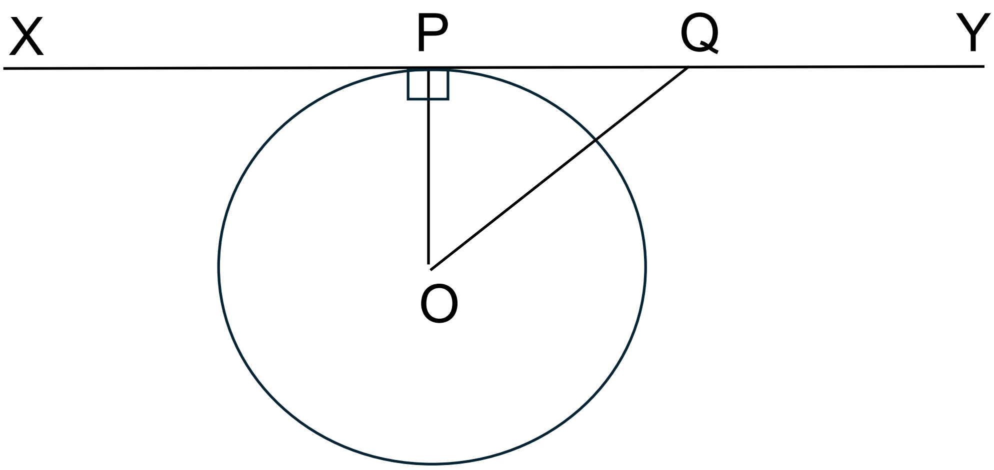
Statement:

The tangent at any point of a circle is perpendicular to the radius drawn through the point of contact.

Explanation:

If a tangent touches the circle at point P and O is the centre, then:

OP ⊥ Tangent



Key Idea:

- Radius is the **shortest distance** from centre to tangent
- Hence, it forms a **right angle (90°)**

Proof of Theorem 1 (Simplified)

Given: XY is tangent at P
To Prove: OP ⊥ XY

Proof:
1. Join OP
2. Take any point Q on XY
3. OQ > OP
4. OP is shortest distance

Thus, OP is shortest distance → perpendicular

✓ Hence proved: **OP ⊥ XY**

Theorem 2: Length of Tangents

Statement:

The lengths of tangents drawn from an external point to a circle are **equal**.

Explanation:

From an external point P, two tangents PA and PB are drawn:

PA = PB

Proof of Theorem 2

- Join OA, OB, OP
- OA = OB (radii)
- OP = common side
- $\angle OAP = \angle OBP = 90°$

Using RHS congruence:

$\Delta OAP \cong \Delta OBP$

$\Rightarrow$ PA = PB

✓ Hence proved

Important Properties of Circles

Chords and Angles

- Equal chords → equal angles at centre
- Equal angles at centre → equal chords

Perpendicular from Centre

- Perpendicular from centre to chord **bisects the chord**
- Line through centre bisecting chord is **perpendicular**

Distance Property

- Equal chords are equidistant from centre
- Chords equidistant from centre are equal

Arc and Angle Properties

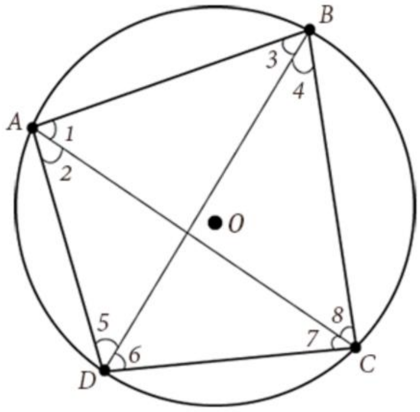
- Equal arcs → equal angles at centre
- Angle at centre = **2 × angle at circumference**

Cyclic Quadrilateral

A quadrilateral is **cyclic** if all its vertices lie on a circle.

Properties:

- Opposite angles sum = **180°**
- If opposite angles sum to 180°, then quadrilateral is cyclic



Important Results

- Only **one circle** can pass through 3 non-collinear points
- Angles in same segment are equal
- Tangents at ends of diameter are parallel

Length of Tangent

The length from external point to point of contact is called **tangent length**.

Formula:

If P is external point:

PA = PB

Important Solved Concepts

Example 1: Tangent Length

If PA = 5 cm, then PB = 5 cm

Example 2: Right Angle Property

If OP = radius and tangent drawn at P:

Angle between OP and tangent = **90°**

Example 3: Cyclic Quadrilateral

If $\angle A + \angle C = 180^\circ$

✓ Quadrilateral is cyclic

Reasoning-Based Concepts

- Tangent always forms **right angle with radius**
- Two tangents from same point are **equal**
- Secant cuts circle, tangent touches

Frequently Asked Questions

Q1: Can a tangent intersect a circle at two points?

✕ No

Q2: How many tangents from a point inside circle?

✓ Zero

Q3: Are tangents equal from external point?

✓ Yes

Real-Life Applications

- Wheels and circular motion
- Engineering design
- Road curves

- Architecture

Short Questions

- Prove tangent ⊥ radius
- Show two tangents are equal
- Identify cyclic quadrilateral

Long Questions

- Prove both theorems
- Solve tangent length problems
- Apply cyclic properties

Learning Outcomes

After completing this chapter, students will be able to:

- Understand circle properties
- Identify tangent and secant
- Apply tangent theorems
- Solve geometry problems
- Use cyclic quadrilateral rules