CHAPTER 4: QUADRATIC EQUATIONS

Introduction

Quadratic equations are one of the most important topics in mathematics and are widely used in real-life situations such as physics, engineering, business calculations, and motion problems.

A quadratic equation represents a relationship where the highest power of the variable is 2. These equations often produce graphs in the shape of a parabola and can have different types of solutions depending on their nature.

Quadratic Polynomial

A polynomial of degree 2 is called a **quadratic polynomial**.

General Form:

$$ax^2 + bx + c$$

Where:

- a, b, c are real numbers
- $a \neq 0$
- a = coefficient of x^2
- b = coefficient of x
- c = constant

Examples:

- $x^2 + 5x + 6$
- $2x^2 - 3x + 1$

Quadratic Equation

When a quadratic polynomial is equated to zero, it becomes a **quadratic equation**.

Standard Form:

$$ax^2 + bx + c = 0$$

Where:

- a= coefficient of x^2(quadratic coefficient)
- b= coefficient of x(linear coefficient)
- c= constant term

Roots of a Quadratic Equation

The values of $x$which satisfy the equation are called **roots** or **solutions**.

If $\alpha$is a root, then:

$$a\alpha^2 + b\alpha + c = 0$$

Methods of Solving Quadratic Equations

There are three main methods:

1. Factorisation Method

2. Completing the Square Method

3. Quadratic Formula

Factorisation Method

This method is used when the quadratic expression can be factorised easily.

Steps:

1. Write equation in standard form
2. Split middle term
3. Factorise
4. Solve by equating each factor to zero

Example:

Solve:

$$2x^2 - 5x + 3 = 0$$

Split middle term:

$$2x^2 - 2x - 3x + 3 = 0$$

Factorise:

$$(x - 1)(2x - 3) = 0$$

Solutions:

$$x = 1, \frac{3}{2}$$

One full solved example step-by-step:

Example:

$x^2 + 5x + 6 = 0$

Split middle term:

$x^2 + 2x + 3x + 6$

Factorise:

x(x + 2) + 3(x + 2)

(x + 2)(x + 3) = 0

Solutions: x = -2, -3

Completing the Square Method

This method converts the equation into a perfect square form.

Steps:

1. Make coefficient of $x^2 = 1$
2. Move constant to RHS
3. Add square of half of coefficient of x
4. Convert into square form

Example:

$$\boldsymbol{x^2 - 4x - 5 = 0}$$

$$x^2 - 4x = 5$$

Add 4:

$$(x - 2)^2 = 9$$

$$x - 2 = \pm 3$$

$$x = 5, \ -1$$

Quadratic Formula

This is the most universal method.

Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:

$$\boldsymbol{x^2 - 5x + 6 = 0}$$

$$a = 1, b = -5, c = 6$$

$$D = 25 - 24 = 1$$

$$x = \frac{5 \pm 1}{2}$$

$$x = 3, 2$$

Discriminant

The expression:

$$D = b^2 - 4ac$$

is called the **discriminant**.

Nature of Roots

Discriminant (D)	**Nature of Roots**
D > 0	Two distinct real roots
D = 0	Two equal real roots
D < 0	No real roots

Graphical Representation

- The graph of a quadratic equation is a **parabola**

Cases:

- Cuts x-axis → two real roots

- Touches x-axis → equal roots
- Does not touch → no real roots

Direction:

- $a > 0$→ opens upward
- $a < 0$→ opens downward

Relationship Between Roots and Coefficients

If roots are α, β:

Formulas:

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Formation of Quadratic Equation

If roots are known:

$$(x - \alpha)(x - \beta) = 0$$
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Example:

Roots = -3, 4

$$x^2 - x - 12 = 0$$

Situational Problems (Word Problems)

Quadratic equations are used in **real-life problems.**

Example 1:

A number increased by 12 equals 160 times its reciprocal.

Let number = x

$$x + 12 = \frac{160}{x}$$

$$x^2 + 12x - 160 = 0$$

Solve → $x = 8$

Example 2:

Speed problem

$$\frac{distance}{speed} + \frac{distance}{new\ speed} = time$$

Forms quadratic equation.

Important Tips & Tricks

- Always convert into standard form first.
- Check discriminant before solving.
- Use factorisation only when easy.
- Use formula for tough questions.

Quick Revision Box

- Standard form → $ax^2 + bx + c = 0$
- Discriminant → $b^2 - 4ac$
- Roots formula → $\frac{-b \pm \sqrt{D}}{2a}$
- Sum → $-b/a$
- Product → c/a

Important Questions for Practice

- Solve using factorisation
- Solve using quadratic formula
- Find nature of roots
- Form equation from roots
- Word problems

Summary

- Quadratic equations are second-degree equations
- Can have 0, 1, or 2 real roots
- Solved using factorisation, completing square, or formula
- Discriminant determines nature of roots
- Used in many real-life situations