



# CHAPTER 5: ARITHMETIC PROGRESSIONS

## Introduction

Mathematics is full of patterns, and recognizing patterns helps us understand real-life situations easily. In our daily life, many situations follow a regular pattern such as salary increments, savings, distances, and arrangements.

For example:

- Salary increasing every year by a fixed amount
- Steps of a ladder decreasing in size
- Saving money regularly with a fixed increase

All these situations follow a special type of pattern called an **Arithmetic Progression (AP)**.

## Sequences, Series and Progressions

Before understanding AP, let us learn some basic terms:

### Sequence

A **sequence** is a list of numbers arranged in a specific order.

Example:

1, 2, 3, 4, 5, ...

Each number is called a **term**.

### Series

A **series** is the **sum of the terms** of a sequence.

Example:

$1 + 2 + 3 + 4 + 5 + \dots$

### Progression

A **progression** is a sequence that follows a definite rule or pattern.



# Arithmetic Progression (AP)

An Arithmetic Progression is a sequence in which the **difference between consecutive terms is constant**.

## Definition

An AP is a list of numbers where each term is obtained by adding a fixed number to the previous term.

## General Form of AP

$a, a + d, a + 2d, a + 3d, \dots$

Where:

- $a$  = first term
- $d$  = common difference

## Example

2, 5, 8, 11, 14, ...

Here:

- First term ( $a$ ) = 2
- Common difference ( $d$ ) = 3

## Common Difference ( $d$ )

The difference between two consecutive terms is called the **common difference**.

**Formula:**

$$d = a_n - a_{n-1}$$

## Types of AP based on $d$

Type	Condition	Example
Increasing AP	$d > 0$	2, 5, 8, 11
Decreasing AP	$d < 0$	10, 7, 4, 1
Constant AP	$d = 0$	3, 3, 3, 3

## Real Life Examples of AP



1. Salary increasing yearly
2. Saving money regularly
3. Ladder steps decreasing in length
4. Rows of plants in a garden
5. Uniform increase in production

## Finite and Infinite AP

### Finite AP

An AP with a fixed number of terms.

Example:

2, 5, 8, ..., 50

### Infinite AP

An AP with no end.

Example:

2, 5, 8, 11, ...

## General Term of AP (nth Term)

The nth term tells us the value of any term without writing all previous terms.

**Formula:**

$$a_n = a + (n - 1)d$$

### Example

Find 10th term of: 2, 7, 12, ...

Here:

- $a = 2$
- $d = 5$
- $n = 10$

$$\begin{aligned} a_{10} &= 2 + (10 - 1) \times 5 \\ &= 2 + 45 \\ &= \mathbf{47} \end{aligned}$$



## Important Applications of nth Term

Given AP → Find a & d → Use formula → Solve

- Finding a particular term
- Checking if a number is a term
- Finding number of terms
- Solving real-life problems

## Finding a Term in AP

To check if a number belongs to an AP:

**Use formula:**

$$a_n = a + (n - 1)d$$

Solve for n

If n is a whole number → term exists

Otherwise → not a term

## Sum of First n Terms of AP

Instead of adding all terms manually, we use a formula.

**Formula 1:**

$$S_n = n/2 [2a + (n - 1)d]$$

**Formula 2 (when last term is known):**

$$S_n = n/2 (a + l)$$

Where:

- l = last term

## Derivation Idea

We write the series forward and backward:

$$S = a + (a+d) + \dots$$

$$S = (\text{last term}) + \dots + a$$



### Adding both:

$$2S = n(\text{first} + \text{last})$$

So,

$$S = n/2 (a + l)$$

### Example (Sum of AP)

Find sum of first 22 terms of: 8, 3, -2, ...

Here:

- $a = 8$
- $d = -5$
- $n = 22$

$$\begin{aligned} S &= 22/2 [2(8) + (21)(-5)] \\ &= 11 [16 - 105] \\ &= 11 \times (-89) \\ &= \mathbf{-979} \end{aligned}$$

## Special Case: Sum of Natural Numbers

$$1 + 2 + 3 + \dots + n$$

**Formula:**

$$S_n = n(n+1)/2$$

### Example

Sum of first 10 numbers:

$$S = 10(11)/2 = \mathbf{55}$$

## Arithmetic Mean (A.M.)

If a, b, c are in AP, then:

$$b = (a + c)/2$$

So, **b is called the Arithmetic Mean** of a and c.



## Important Properties of AP

1. Difference between consecutive terms is constant
2. Sum of equidistant terms is equal

Example:

$$2 + 17 = 5 + 14$$

## Daily Life Applications of AP

- Salary increment problems
- Interest calculations
- Savings schemes
- Production growth
- Construction and design
- Distance and speed problems

## Common Mistakes to Avoid ✕

- Using wrong formula
- Confusing  $n$  and  $a_n$
- Incorrect sign of  $d$
- Forgetting brackets
- Wrong substitution

## Quick Revision Formula Box

- General form:  $a, a+d, a+2d\dots$
- Common difference:  $d = a_n - a_{n-1}$
- $n$ th term:  $a_n = a + (n-1)d$
- Sum formula:  $S_n = \frac{n}{2} [2a + (n-1)d]$
- Sum (with last term):  $S_n = \frac{n}{2} (a + l)$
- Natural numbers:  $S_n = \frac{n(n+1)}{2}$
- A.M:  $b = \frac{a + c}{2}$

## Important Question Types

- Find  $n$ th term



- Find number of terms
- Check if a number is in AP
- Find sum of AP
- Word problems (salary, savings, etc.)

## Learning Outcomes

After studying this chapter, students will be able to:

- Understand patterns in real life
- Identify Arithmetic Progressions
- Calculate nth term easily
- Find sum of series quickly
- Apply formulas in real-life situations