



CHAPTER 8: INTRODUCTION TO TRIGONOMETRY

Introduction

Trigonometry is an important branch of mathematics that deals with the relationship between the **angles and sides of a triangle**, especially a **right-angled triangle**.

The word *Trigonometry* is derived from:

- **Tri** → Three
- **Gon** → Sides
- **Metron** → Measure

So, trigonometry literally means “**measurement of triangles**.”

Real-Life Applications

Trigonometry is used in:

- Measuring heights of buildings (without directly measuring)
- Finding distance across rivers
- Navigation, astronomy, engineering, and physics

Example:

- Finding height of a tower using shadow
- Calculating distance of a flying object

Trigonometric Ratios

Consider a **right-angled triangle ABC**, right-angled at B.

For angle **A**:

- **Opposite side** → BC
- **Adjacent side** → AB
- **Hypotenuse** → AC (longest side)

Six Trigonometric Ratios

- $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$



- $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$
- $\operatorname{cosec} A = \frac{1}{\sin A} = \frac{AC}{BC}$
- $\sec A = \frac{1}{\cos A} = \frac{AC}{AB}$
- $\cot A = \frac{1}{\tan A} = \frac{AB}{BC}$

Important Relationships

- $\tan A = \frac{\sin A}{\cos A}$
- $\cot A = \frac{\cos A}{\sin A}$
- $\operatorname{cosec} A = \frac{1}{\sin A}$
- $\sec A = \frac{1}{\cos A}$

Key Observation

Trigonometric ratios depend **only on the angle**, not on the size of the triangle.
Even if triangle size changes, ratios remain same for same angle.

Using Pythagoras Theorem

If one ratio is given, we can find others using:

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

Example

If

$$\tan A = \frac{4}{3}$$



Then:

- Opposite = $4k$
- Adjacent = $3k$

Using Pythagoras:

$$AC = 5k$$

So:

$$\sin A = \frac{4}{5},$$

$$\cos A = \frac{3}{5}$$

8.3 Trigonometric Ratios of Specific Angles

We calculate values for:

- $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$

Values Table

ANGLE	SIN	COS	TAN	COSEC	SEC	COT
0°	0	1	0	ND	1	ND
30°	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}$
45°	$1/\sqrt{2}$	$1/\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2/\sqrt{3}$	2	$1/\sqrt{3}$
90°	1	0	ND	1	ND	0

ND = Not Defined

Important Observations

- $\sin \theta$ increases from **0 to 1**
- $\cos \theta$ decreases from **1 to 0**
- $\tan \theta$ increases from **0 to ∞**

Special Angle Logic



- ✓ 45° → Isosceles triangle
- ✓ 30° & 60° → Equilateral triangle split
- ✓ 0° & 90° → Limit concept

8.4 Trigonometric Identities

An identity is an equation true for all values.

Main Identities

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

Derived Forms

$$\cos^2 A = 1 - \sin^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

8.5 Complementary Angles

If two angles add up to 90° , they are complementary.

Formulas

$$\sin(90^\circ - A) = \cos A$$

$$\cos(90^\circ - A) = \sin A$$

$$\tan(90^\circ - A) = \cot A$$

$$\sec(90^\circ - A) = \operatorname{cosec} A$$

Range of Trigonometric Ratios

For $0^\circ \leq \theta \leq 90^\circ$:

- $0 \leq \sin \theta \leq 1$



- $0 \leq \cos \theta \leq 1$
- $0 \leq \tan \theta < \infty$

Important Notes

- ✓ Hypotenuse is always the longest side
- ✓ $\sin A$ and $\cos A$ are always ≤ 1
- ✓ $\sec A$ and $\operatorname{cosec} A \geq 1$

Solved Examples

Example 1

$$\sin A = 4/5$$

Solution:

$$\text{Opp} = 4k, \quad \text{Hyp} = 5k$$

Using Pythagoras:

$$\text{Adj} = 3k$$

So,

$$\cos A = 3/5$$

$$\tan A = 4/3$$

Example 2

$$\text{If } \cot A = \frac{8}{15}$$

Then:

$$\tan A = \frac{15}{8}$$

Using Pythagoras:

$$\sin A = \frac{15}{17}$$

$$\sec A = \frac{17}{8}$$



Example 3

Find value of:

$$\begin{aligned} & \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$$

Key Takeaways (VERY IMPORTANT)

- Trigonometry deals with **angles & sides**
- Always identify:
 - Opposite
 - Adjacent
 - Hypotenuse
- Learn **standard values table**
- Practice identities regularly