

CHAPTER 6: LINES AND ANGLES

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

Introduction and Overview

Lines and angles form the backbone of geometry. This chapter builds a foundation in understanding types of lines, angle properties, and their interrelationships. Mastery of this chapter is vital for higher studies in geometry.

Section 1: Basic Terms Related to Lines and Angles

1.1 Definitions

- **Line:** Extends infinitely in both directions.
- **Line segment:** Part of a line with two endpoints.
- **Ray:** Extends infinitely in one direction from its endpoint.
- **Point:** Location with no size.
- **Collinear Points:** Points on the same line.
- **Non-Collinear Points:** Points not on the same line.

Angle Type	Description	Measure
Acute Angle	Angle less than 90°	$0^\circ < \theta < 90^\circ$
Right Angle	Exactly 90° angle	$\theta = 90^\circ$
Obtuse Angle	Greater than 90° but less than 180°	$90^\circ < \theta < 180^\circ$
Straight Angle	Exactly 180° angle	$\theta = 180^\circ$
Reflex Angle	Between 180° and 360°	$180^\circ < \theta < 360^\circ$
Complete Angle	Exactly 360° , a full turn around a point	$\theta = 360^\circ$

1.2 Angles and Their Types

An angle is formed by two rays with a common endpoint called the vertex.

Section 2: Pairs of Angles

2.1 Adjacent Angles

Two angles sharing a vertex and one arm, without overlapping.

2.2 Linear Pair of Angles

- Formed when two adjacent angles have non-common arms forming a straight line.

- Sum is always 180° (Linear Pair Axiom).
- Converse is equally important: if sum is 180° and angles are adjacent, they form a linear pair.

2.3 Vertically Opposite Angles

- When two lines intersect, the angles opposite each other are equal.

2.4 Complementary and Supplementary Angles

- Complementary angles: Sum to 90° .
- Supplementary angles: Sum to 180° .

Section 3: Intersecting and Parallel Lines

3.1 Intersecting Lines

Two lines that meet at a point.

3.2 Parallel Lines

Lines in the same plane which never intersect.

3.3 Transversal

A line that cuts two or more lines.

3.4 Angles Formed by Parallel Lines and a Transversal

- Corresponding angles are equal.
- Alternate interior angles are equal.

- Alternate exterior angles are equal.
- Consecutive interior angles are supplementary.

Section 4: Important Theorems and Proofs

4.1 Linear Pair Axiom Proof

Proof shows that adjacent angles formed by a ray on a straight line sum to 180° .

4.2 Vertically Opposite Angles Theorem

Opposite angles, when two lines intersect, are equal.

4.3 Corresponding Angles Postulate and Its Converse

If two lines are parallel and cut by a transversal, corresponding angles are equal. Conversely, if corresponding angles are equal, lines are parallel.

4.4 Alternate Interior and Exterior Angles Theorems

Equal angles on alternate sides of the transversal, inside or outside parallel lines.

4.5 Angle Sum Property of a Triangle

Sum of interior angles in a triangle = 180° . Used to solve multiple problems and is foundational for polygon properties.

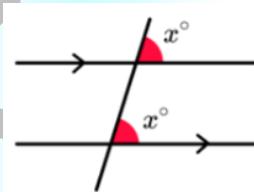
Section 5: Solved NCERT Questions with Detailed Solutions

Question 1:

Two lines intersect each other. One of the angles formed is 120° . Find the other three angles formed.

Solution:

- Vertically opposite angle = 120° .
- Adjacent angles form a linear pair \rightarrow sum to 180° .
- Adjacent angles = $180^\circ - 120^\circ = 60^\circ$.
- The angle vertically opposite to 60° is also 60° .
- Hence, the angles formed are $120^\circ, 60^\circ, 120^\circ, 60^\circ$.

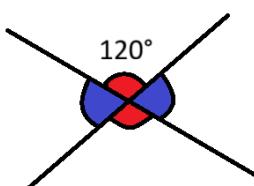


Question 2:

In the figure, a transversal intersects two lines such that a pair of corresponding angles are equal. Prove that the two lines are parallel.

Solution:

- According to the corresponding angles converse theorem, if a transversal cuts two lines and the corresponding angles are equal, then those two lines are parallel.
- If lines were not parallel, these corresponding angles would not be equal, which contradicts the given information.
- Therefore, the two lines must be parallel.



Question 3:

Prove that the bisectors of the angles of a linear pair are perpendicular to each other.

Solution:

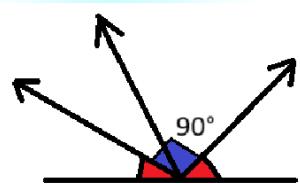
- Let the linear pair be $\angle AOB$ and $\angle BOC$ such that $\angle AOB + \angle BOC = 180^\circ$.
- The bisector of $\angle AOB = \frac{1}{2}\angle AOB$, and that of $\angle BOC = \frac{1}{2}\angle BOC$.
- Sum of the bisected angles $= \frac{1}{2} \times 180^\circ = 90^\circ$.
- Hence, the bisectors are perpendicular.

Question 4:

Two parallel lines are cut by a transversal. The consecutive interior angles are 70° and $(3x + 5)^\circ$. Find x and the angles.

Solution:

- Consecutive interior angles are supplementary:
 $70^\circ + (3x + 5)^\circ = 180^\circ$.
- $3x + 75 = 180 \Rightarrow 3x = 105 \Rightarrow x = 35$.
- The other angle $= 3 \times 35 + 5 = 110^\circ$.
- Thus, the angles are 70° and 110° .



Question 5:

Prove that the sum of the exterior angles of any polygon is 360° .

Solution:

- Each exterior angle forms a linear pair with the interior angle at every vertex.
- Sum of interior angles of an n -sided polygon = $(n - 2) \times 180^\circ$.
- Sum of all exterior angles = $n \times 180^\circ - (n - 2) \times 180^\circ = 360^\circ$.
- Therefore, the sum of all exterior angles of any polygon is 360° .

Section 6: Practice Drill (Consolidated)

- Find unknown angles given partial angle measures in intersecting lines.
- Prove angle equalities for various pairs formed by a transversal.
- Solve complex problems combining vertical angles, linear pairs, and parallel line properties.
- Exercise on polygon exterior angles using triangle sum properties.

Section 7: Common Errors and How to Avoid Them

- Confusing angle pairs and their properties; memorize definitions clearly.

- Forgetting to verify if lines are parallel before applying properties of transversal.
- Incorrect notation of angle vertices leading to wrong angle identification.
- Misinterpreting linear pair as just adjacent angles without the straight line condition.
- Neglecting converse theorems in proofs.
- Mixing complementary and supplementary angles.
- Not drawing clear, accurate diagrams.

Section 8: Summary of Important Formulas & Properties

Concept	Property/Formula
Linear Pair Angles	Sum = 180°
Vertically Opposite Angles	Equal
Corresponding Angles	Equal when lines are parallel
Alternate Interior Angles	Equal when lines are parallel
Alternate Exterior Angles	Equal when lines are parallel
Consecutive Interior Angles	Sum to 180° when lines are parallel
Angle Sum of Triangle	180°
Exterior Angle of Triangle	Equal to the sum of opposite interior angles
Sum of Exterior Angles Polygon	360°

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