CHAPTER 10: HERON'S FORMULA

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

1. Historical Context & Why It Matters

Hero, a Greek mathematician in Alexandria's famous library, needed to measure irregular land plots without heights. His formula bypassed rulers and protractors, using only perimeter data. Today, it powers GPS mapping, construction blueprints, and even video game physics engines.

Real-World Impact:

- **Land surveyors calculate farm plots (sides measured by walking)**
- **Architects design triangular roof trusses**
- **Game developers render 3D terrain**

2. Core Formula & Notation

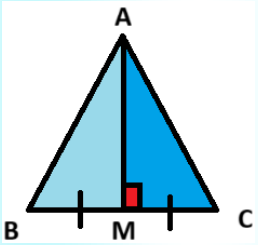
For triangle ABC with sides a (BC), b (AC), c (AB):

Semi-perimeter: $s = \frac{a+b+c}{2}$

Area: $A = \sqrt{s(s-a)(s-b)(s-c)}$

Visual Breakdown: Imagine s as "half the boundary walk." Each $s - side$ represents excess perimeter opposite that side.

3. Geometric Proof



Construction: Drop perpendicular from B to AC at M, splitting AC into m and n (where $a = m + n$).

Pythagoras applies:

$$h^2 = b^2 - m^2 = c^2 - n^2$$
$$b^2 - c^2 = m^2 - n^2 = (m - n)(m + n) = (m - n)a$$
$$m - n = \frac{b^2 - c^2}{a}$$

Solve system:

$$m = \frac{2b^2 + a^2 - c^2}{4a}, n = \frac{2c^2 + a^2 - b^2}{4a}$$

Area $A = \frac{1}{2}ah$, **substitute** $h = \sqrt{b^2 - m^2}$, **simplify algebraically:** $16A^2 = 4a^2h^2 = (2bh)^2 - (b^2 - c^2 + a^2)^2/a^2$ **(full expansion yields Heron form).**

Mnemonic: Product $s(s - a)(s - b)(s - c)$ balances perimeter contributions perfectly.

4. Special Cases Derived

Equilateral (sides a**):** $s = \frac{3a}{2}$

$$A = \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3} = \frac{\sqrt{3}}{4}a^2$$

Isosceles (sides b, b, a): Simplifies to height formula verification.

Right Triangle Check: Sides 3-4-5, $s = 6$, $A = \sqrt{6(3)(2)(1)} = 6$ matches $\frac{1}{2} \times 3 \times 4$.

5. 5-Step Mastery Process

1. LIST sides: a=__, b=__, c=__

2. COMPUTE s = (a+b+c)/2

3. FORM terms: s-a, s-b, s-c

4. PRODUCT: s × (s-a) × (s-b) × (s-c)

5. √PRODUCT = Area (units²)

Pro Tip: Prime factorize product for exact roots (e.g., 144=12²).

6. Elite Solved Examples

Level 1: Basic Scalene

Q1. Sides are 5cm, 6cm, 7cm. What is the area of the triangle?

Sol.

s=9, A=√[9(4)(3)(2)]=√216=6√6 cm² ≈14.70

Level 2: Equilateral

Q2. Side is 10cm. Find the area of the triangle.

Sol.

A=(√3/4)(100)=25√3 cm² ≈43.30

Level 3: Word Problem

Q3. Traffic sign triangle: sides 50cm, 60cm, 70cm. Find the Area of the triangle.

Sol.

$s=90$, $A=\sqrt{[90(40)(30)(20)]}=\sqrt{2,160,000}=1469.69\ cm^2$

Level 4: Ratio Sides

Q4. Ratio of sides is3:4:5 and perimeter is 120cm. Find the area of triangle.

Sol.

Ratio of sides = 3 : 4 : 5

Let sides be 3x, 4x, 5x

Perimeter = 3x + 4x + 5x = 120

12x = 120

x = 10

Actual sides = 30 cm, 40 cm, 50 cm (right-angled triangle)

Area = $1/2 \times 30 \times 40 = 600\ cm^2$

Level 5: Isosceles Challenge

Q5. Legs 13m, base 10m (park slide)

$s=18$, $A=\sqrt{[18(5)(5)(8)]}=\sqrt{3600}=60m^2$

Lightning Drills (Solve in 90 seconds):

- **Perimeter 84 cm, ratio 2:3:4 → Find sides, area?**
- **Equilateral area $100\sqrt{3}\ cm^2$ → Side length?**

7. Error Traps & Fixes (Exam Savers)

Deadly Mistake	Root Cause	Elite Fix
s = a+b+c (no /2)	Formula amnesia	Write formula FIRST every time
s-a negative	Invalid triangle	Triangle Inequality: Sort x≤y≤z, x+y>z
Premature rounding	√21.6 ≈ 4.65 error	Keep 6√6 exact until end
Mixed units	cm + m chaos	Convert ALL to cm or m
Zero area	Collinear points	Verify s > each side

Triangle Inequality Law: For sides $x \le y \le z$, must satisfy $x + y > z$, $x + z > y$, $y + z > x$.

8. Triangle Type Arsenal

Type	Side Pattern	Fast Formula	Example Area
Scalene	All different	Full Heron	14.7 cm²
Equilateral	a=a=a	$(\sqrt{3}/4)a^2$	43.3 cm²
Isosceles	b=b,a	Heron or $\frac{1}{2}a\sqrt{(b^2-(a/2)^2)}$	60 m²
Right	a,b,$\sqrt{(a^2+b^2)}$	½ab (verifies Heron)	6 units²

9. Self-Challenge Quiz (Score 100%)

1. **Semi-perimeter for sides 9-10-17 cm?**
2. **Area of triangle with sides 3-4-5 cm?**
3. **Invalid triangle sides?**

4. Equilateral triangle side whose area is $100\sqrt{3}$ cm²?
5. Perimeter is 50 cm and two equal sides are 13 cm each. Find third side & area?

10. Premium Glossary

- Semi-perimeter (s): Half the perimeter walk
- Triangle Inequality: Foundation of triangle existence
- Radical (√): Square root extracts area perfectly

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