CHAPTER 4: LINEAR EQUATIONS IN TWO VARIABLES

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

1. Introduction to Linear Equations in Two Variables

A linear equation in two variables is an algebraic equation that can be written in the form:

$$ax + by + c = 0$$

where a, b, and c are real numbers and at least one of a or b is not zero. The variables x and y represent unknowns.

- a and b cannot both be zero simultaneously.
- Both x and y are raised to the power of 1.
- These equations represent a line on a coordinate plane.
- Unlike equations in one variable, such equations have infinitely many solutions.
- Each solution is an ordered pair (x, y) that satisfies the equation.

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2. Understanding Solutions of Linear Equations in Two Variables

- A solution is a pair (x, y) that, when substituted into the equation, satisfies it.
- To find solutions, assign arbitrary values to one variable and solve for the other.
- Since there are infinitely many such pairs, solutions form a set represented by points on a graph.

Example 1:

Equation: $2x + 3y = 6$

- If $x = 0$, then $3y = 6 \Rightarrow y = 2$
- If $y = 0$, then $2x = 6 \Rightarrow x = 3$

Solutions: $(0, 2)$, $(3, 0)$

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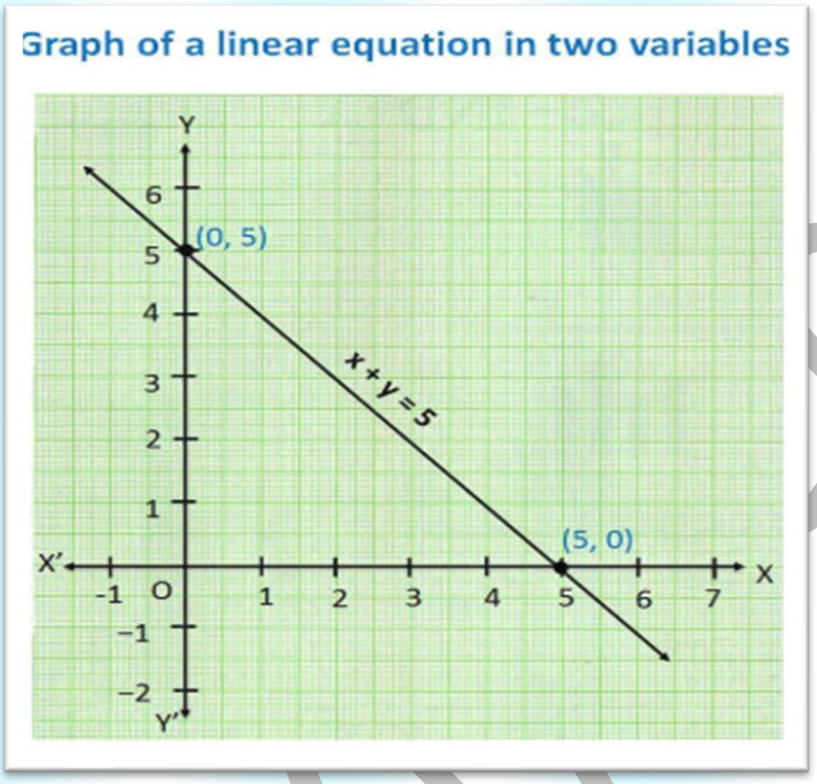
3. Graphical Representation

- Plotting the solutions as points on the Cartesian plane creates a straight line.
- This line is called the graph of the equation.
- To plot the graph:

1. Find at least two solutions (points).
2. Plot these points on the plane.

3. **Draw a straight line through them (extend in both directions).**



Vertical and Horizontal Lines:

- **An equation like $x = k$ represents a vertical line passing through $x = k$.**
- **An equation like $y = k$ represents a horizontal line passing through $y = k$.**

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4. Properties of Linear Equations

- **Adding, subtracting, multiplying, or dividing both sides by the same number (except zero) does not change the solution set.**
- **Two linear equations represent lines that may:**
  - **Intersect at exactly one point (one unique solution).**
  - **Coincide (infinitely many solutions).**
  - **Be parallel (no solution).**

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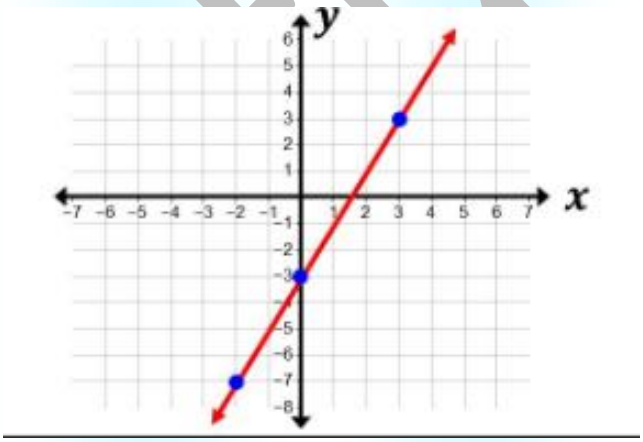
5. Methods to Solve Systems of Linear Equations in Two Variables

1. Graphical Method:

Plot both equations and find the intersection point(s).



2. Substitution Method:

Solve one equation for one variable and substitute into the other to find the other variable.

$x - 2y = 8$ ← (1)

$x + y = 5$ ← (2)

From equation (2),

$x = 5 - y$

Substitute this value in equation (1),

$x - 2y = 8$

$5 - y - 2y = 8$

$5 - 3y = 8$

$-3y = 8 - 5$

$-3y = 3$

$y = -1$

Put this value in equation (2),

$x - 1 = 5$

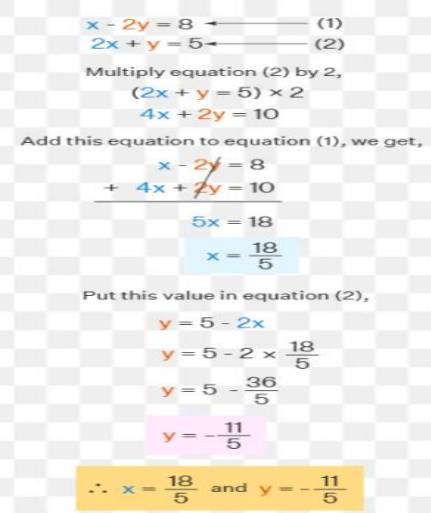
$x = 5 + 1$

$x = 6$

$\therefore x = 6$ and $y = -1$

3. Elimination Method:

Manipulate equations to eliminate one variable, then solve for the remaining variable.



Step-by-Step Examples

Example 2 (Substitution):

$$3x + 2y = 12 (1) x - y = 3 (2)$$

From (2), express $x = y + 3$. Substitute into (1):

$$3(y + 3) + 2y = 123y + 9 + 2y = 125y = 3y = \frac{3}{5}$$

Then,

$$x = \frac{3}{5} + 3 = \frac{18}{5}$$

Solution: $(\frac{18}{5}, \frac{3}{5})$

Example 3 (Elimination):

$$2x + 3y = 64x - 3y = 12$$

Add the two equations:

$$6x = 18 \Rightarrow x = 3$$

Substitute $x = 3$ into first equation:

$$2(3) + 3y = 6 \Rightarrow 6 + 3y = 6 \Rightarrow y = 0$$

Solution: $(3, 0)$

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5. Graph Plotting Stepwise

For $x + y = 5$:

- **Let $x = 0 \Rightarrow y = 5$**
- **Let $y = 0 \Rightarrow x = 5$**

  Plot points $(0, 5)$ and $(5, 0)$ and draw the line through them.

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6. Special Cases

- Vertical lines: $x = k$
- Horizontal lines: $y = k$

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7. Practice Questions

1. Find three solutions and plot for $3x - 4y = 8$.
2. Solve by substitution:

$$x + y = 102x - y = 3$$

3. Graphically represent the solutions for $x = 7$ and $y = -2$.

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