



CHAPTER 3: PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Introduction

In real life, we often come across situations where two unknown quantities are related to each other.

For example:

Finding number of items, cost, age problems, etc.

Such situations can be represented using **pair of linear equations in two variables**.

In this chapter, we will study:

- Graphical method of solving equations
- Algebraic methods (Substitution and Elimination)
- Nature of solutions (consistent/inconsistent)
- Word problems

Linear Equation in Two Variables

A **linear equation** in two variables is written in the form:

$$ax + by + c = 0$$

Where:

- x and y are variables
- a , b , c are constants
- a and b cannot both be zero

Solution of a Linear Equation

A solution of a linear equation is a pair of values (x, y) which satisfies the equation.

Example:

$$\text{For } 2x + y = 4$$

$(0, 4)$ is a solution because it satisfies the equation.

A linear equation has **infinitely many solutions**.



Pair of Linear Equations

Two linear equations taken together are called a **pair of linear equations**.

General form:

- $a_1x + b_1y + c_1 = 0$
- $a_2x + b_2y + c_2 = 0$

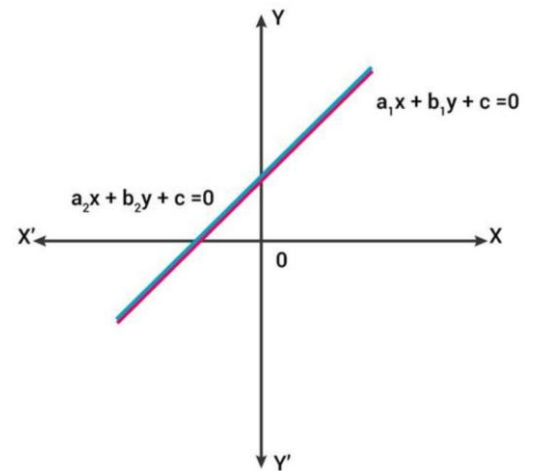
Graphical Representation

Each linear equation represents a **straight line** on a graph.

To draw a graph:

- Take two values of x
- Find corresponding y values
- Plot points and join them

Every point on the line satisfies the equation.



Graphical Method of Solving

Choose values → Find points → Plot → Join → Intersection

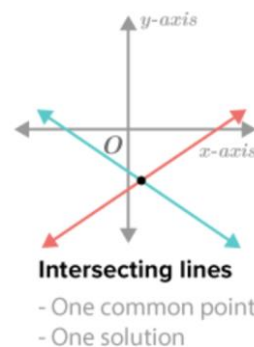
Steps:

1. Draw both lines on graph
2. Find point of intersection
3. That point is the solution

Nature of Solutions (Important)

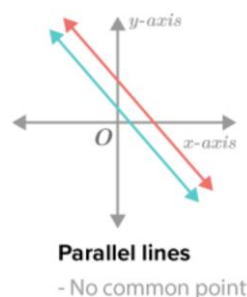
1. Intersecting Lines

- Lines meet at one point
- One unique solution
- Consistent pair



2. Parallel Lines

- Lines never meet

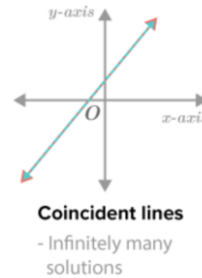




- No solution
- Inconsistent pair

3. Coincident Lines

- Lines overlap completely
- Infinite solutions
- Dependent pair



Algebraic Conditions for Solutions

For equations:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

If

- $a_1/a_2 \neq b_1/b_2 \rightarrow$ **Unique solution**

If

- $a_1/a_2 = b_1/b_2 \neq c_1/c_2 \rightarrow$ **No solution**

If

- $a_1/a_2 = b_1/b_2 = c_1/c_2 \rightarrow$ **Infinite solutions**

Graph Trick

- Intersect \rightarrow one point \rightarrow one solution
- Parallel \rightarrow never meet \rightarrow no solution
- Same line \rightarrow overlap \rightarrow infinite solutions

Algebraic Methods of Solving

Graph method is not always accurate, so we use algebraic methods.



1. Substitution Method

Steps:

1. Express one variable in terms of another
2. Substitute into second equation
3. Solve to get one variable
4. Substitute back to find the other

Make x or y → Substitute → Solve → Back substitute

Example:

$$x + y = 5$$

$$x - y = 1$$

From first: $x = 5 - y$

Put in second: $5 - y - y = 1$

$$\Rightarrow 5 - 2y = 1$$

$$\Rightarrow y = 2$$

Now $x = 3$

Solution = $(3, 2)$

2. Elimination Method

Steps:

1. Make coefficients of one variable equal
2. Add or subtract equations
3. One variable gets eliminated
4. Solve for remaining variable

Equal coefficients → Add/Subtract → Solve → Substitute

Example:

$$x + 2y = 8$$

$$2x - 3y = 2$$

Multiply first by 2:

$$2x + 4y = 16$$



Subtract:

$$7y = 14$$

$$\Rightarrow y = 2$$

$$x = 4$$

Solution = (4, 2)

3. Cross Multiplication Method

For equations:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Formula:

$$x = (b_2c_1 - b_1c_2) / (a_1b_2 - a_2b_1)$$

$$y = (c_1a_2 - c_2a_1) / (a_1b_2 - a_2b_1)$$

Useful for direct solving in exams

Special Cases in Algebraic Method

If you get:

- **TRUE** statement (like $18 = 18$) → Infinite solutions
- **FALSE** statement (like $0 = 5$) → No solution

Word Problems

Steps to solve:

1. Assume variables
2. Form equations from conditions
3. Solve using any method
4. Write final answer clearly

Example:

Cost of 5 pens and 7 pencils = ₹50

Cost of 7 pens and 5 pencils = ₹65

Let:

x = cost of pen

y = cost of pencil



Equations:

$$5x + 7y = 50$$

$$7x + 5y = 65$$

Solve to get values.

Reducible Equations

Some equations are not linear but can be converted.

Example:

$$2/x + 3/y = 4$$

Let:

$$1/x = u$$

$$1/y = v$$

Then:

$$2u + 3v = 4$$

Solve normally.

Important Result

A pair of linear equations can be solved by:

- Graphical Method
- Substitution Method
- Elimination Method

Summary

- Linear equation \rightarrow straight line
- Pair of equations \rightarrow two lines
- Solution \rightarrow point of intersection
- Can have:
 - One solution
 - No solution
 - Infinite solutions
- Algebraic methods are more accurate than graphs
- General form: $ax + by + c = 0$
- Solution = point satisfying both equations
- Graph = straight line



- Intersection point = solution
- Degree always = 1