# CHAPTER 2: POLYNOMIALS

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS



## 1. Introduction: Polynomials & Their Importance

A polynomial is an algebraic expression with variables and coefficients, using positive integer exponents only.

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

- Only whole number exponents (not negative, not fractions).
- Used in science, engineering, economics, computers.

### Example:

$4x^3 + 5x^2 - 7x + 10$

### StudyBee Insight:

Understand polynomials to crack tough questions in Boards—and for real-life modeling (like profit, distance, or speed formulas).

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## 2. Terminology: Terms, Degree, and Coefficient

- Term: Product of coefficient and variable(s) (like $3x^2$).

- **Coefficient: Number multiplying the variable (3 in $3x^2$).**
- **Degree: Largest exponent of variable in the polynomial (degree = 2 for $3x^2 + 8$).**

### Table Example:

| Expression | Terms | Coefficients | Degree |
|---|---|---|---|
| $5x^2 + 7x + 8$ | $5x^2$, $7x$, $8$ | 5, 7, 8 | 2 |

## 3. Types of Polynomials (by Terms & Degree)

### By Number of Terms:

- **Monomial: One term (7x)**
- **Binomial: Two terms (x + 5)**
- **Trinomial: Three terms ($x^2 + 3x + 2$)**
- **Polynomial: Four or more terms**

### By Degree:

- **Constant: Degree 0 (3)**
- **Linear: Degree 1 (2x + 1)**
- **Quadratic: Degree 2 ($x^2 + 4x + 4$)**
- **Cubic: Degree 3 ($x^3 - 8$)**

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## 4. Zero and Constant Polynomials

- **Zero Polynomial:** All coefficients zero (0), undefined degree.
- **Constant Polynomial:** Only constant term, no variable (7), degree 0.

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## 5. Algebraic Identities (All NCERT – 9 Major)

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $a^2 - b^2 = (a - b)(a + b)$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
6. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
7. $(x + a)(x + b) = x^2 + (a + b)x + ab$
8. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
9. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

**Each identity is explained with step-by-step expansion, common errors, and application examples.**

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## 6. Value of a Polynomial

**For p(x), substitute value to get p(k). Example:**

**If $p(x) = x^2 - 5x + 6$, then $p(3) = 3^2 - 5\times3 + 6 = 9 - 15 + 6 = 0$.**

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## 7. Zeroes (Roots) of Polynomials

**Definition: Value of x for which p(x) = 0.**

### Linear Example:

**p(x) = 2x + 3, solve 2x + 3 = 0 → x = -1.5.**

### Quadratic Example:

**p(x) = $x^2$ - 2x - 3, factor: (x - 3)(x + 1) = 0 → x = 3, -1.**

### Cubic Example:

**Find zeroes of $x^3$ - $6x^2$ + 11x - 6.**

**Try x = 1: 1 - 6 + 11 - 6 = 0 → x = 1 root.**

**Divide by (x - 1), get quadratic $x^2$ - 5x + 6.**

**Factor: (x - 2)(x - 3).**

**Zeroes: x = 1, 2, 3.**

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## 8. Operations: Addition, Subtraction, Multiplication, Division

**Addition/Subtraction: Combine like terms.**

### Example:

**(3$x^2$ + 4x - 2) + (5$x^2$ - 6x + 8) = 8$x^2$ - 2x + 6**

**Multiplication:** Use distributive property.

**Example:**

$(x + 2)(x^2 - 4x + 3) = x^3 - 4x^2 + 3x + 2x^2 - 8x + 6 = x^3 - 2x^2 - 5x + 6$

**Division:** Use long division, subtract stepwise remainders.

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## 9. Factorization of Polynomials

**Quadratic Factorization:**

$x^2 + 7x + 10 = (x + 5)(x + 2)$

**Cubic Factorization:**

Example 1: $x^3 - 8$

$= x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)$

Example 2: $x^3 - 4x^2 - 7x + 10$

Try $x = 1$: $1 - 4 - 7 + 10 = 0 \rightarrow$ root $x = 1$

Divide by $(x - 1)$ to get quotient $x^2 - 3x - 10$

Factor quotient: $(x - 5)(x + 2)$

Factorization: $(x - 1)(x - 5)(x + 2)$

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## 10. Remainder & Factor Theorems

**Remainder Theorem:** Remainder when $p(x)$ divided by $(x - a)$ is $p(a)$.

**Factor Theorem:** If $p(a) = 0$, $(x - a)$ is a factor.

**Example:**

**Divide $p(x) = x^3 - 3x^2 + 5x - 3$ by $(x - 1)$**

**$p(1) = 1 - 3 + 5 - 3 = 0 \rightarrow (x - 1)$ is factor.**

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## 11.Advanced Concepts: Common Errors, HOTS, Applications

### Common Errors:

- **Using negative or fractional exponents (not allowed in polynomials)**
- **Confusing degree with number of terms**
- **Forgetting middle terms in expansions**

### HOTS:

- **Why at most three roots for cubic?**
- **Create polynomial for points (1,2), (2,5)**
- **Adjust polynomial for double root at x=2**

### Applications:

- **Physics (projectile motion)**
- **Economics (cost/profit functions)**
- **Data Science (regression models)**

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## 12.Practice Problems

### Short Answer:

- Define degree and coefficient with example.
- Give examples: monomial, binomial, polynomial.

### Standard Board Questions:

1. Factorize $x^2 - 2x - 8$.
2. Find zeroes of $x^3 - 6x^2 + 11x - 6$.
3. Expand $(a - b)^3$ using identities.

### Advanced:

1. Factorize $p(x) = x^3 + 2x^2 - 5x + 6$ using Factor Theorem.
2. Find remainder of $2x^3 - 7x^2 + 6$ divided by $(x - 2)$.
3. Write cubic polynomial with zeroes 2, -3, 4.

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