CHAPTER 2: POLYNOMIALS

Introduction

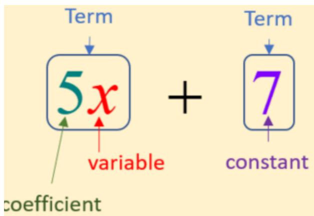
In earlier classes, we studied algebraic expressions and polynomials. In this chapter, we will focus on:

- Zeroes of a polynomial
- Graphical meaning of zeroes
- Relationship between zeroes and coefficients

These concepts are important for understanding equations and solving higher-level problems.

Algebraic Expressions

An **algebraic expression** is a combination of variables, constants, and mathematical operations.



Examples:

- $3x$
- $5x^2 + 2x + 1$
- $4xy – 7$

Terms:

Each part separated by + or – is called a term.

- A term is formed by multiplying constants and variables together.
- An algebraic expression is made up of one or more such terms.

Coefficient:

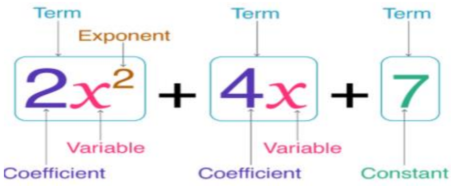
The numerical value multiplying a variable is called its coefficient.

Example, in 4xy, the coefficient is 4

Polynomial

A **polynomial** is a special type of algebraic expression where:

- Powers of variables are whole numbers
- No variable appears in denominator



Examples:

- $3x^2 + 2x + 1$
- $x^3 – 4x$
- 5

Not a Polynomial:

- $1/x$
- $\sqrt{x}$
- x^{-1}

Types of Polynomials (Based on Number of Terms)

Monomial: A polynomial with only one term (e.g., $5x$)
Binomial: A polynomial with two terms (e.g., $x + 3$)
Trinomial: A polynomial with three terms (e.g., $x^2 + 2x + 1$)

Degree of a Polynomial

The highest power of the variable in a polynomial is called its degree.

Examples:

- $2x + 3$ → Degree = 1
- $x^2 – 5x + 6$ → Degree = 2
- $x^3 + x^2 + 1$ → Degree = 3

Types of Polynomials (Based on Degree)

Type	Degree	Example
Linear	1	$2x + 3$
Quadratic	2	$x^2 + 5x + 6$
Cubic	3	$x^3 – 2x$

Value of a Polynomial

If p(x) is a polynomial, then replacing x with a number gives its value.

Example:

$p(x) = x^2 - 2x$
$p(2) = 4 - 4 = 0$

Zeroes of a Polynomial

A number k is called a zero of a polynomial p(x) if:

$p(k) = 0$

Example:

$p(x) = x^2 - 5x + 6$
$p(2) = 0 \rightarrow$ so 2 is a zero
$p(3) = 0 \rightarrow$ so 3 is also a zero

Geometrical Meaning of Zeroes

Zeroes of a polynomial are the **x-coordinates where the graph intersects the x-axis**.

Graph of x^n (Important Concept)

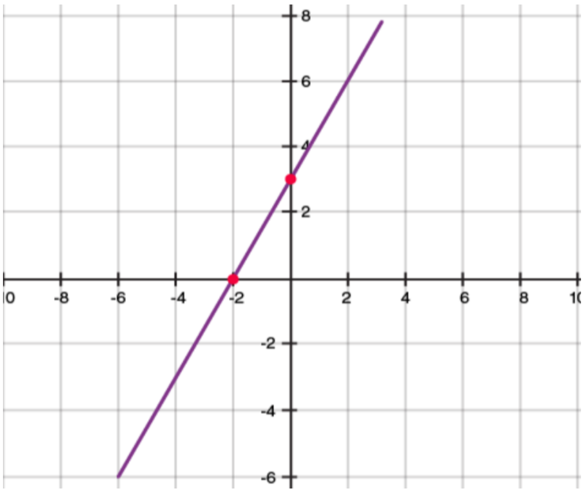
If the power (n) is even, the graph is symmetric and U-shaped.

If the power (n) is odd, the graph passes through opposite quadrants.

The graph of $-x^n$ is the reflection of x^n across the x-axis.

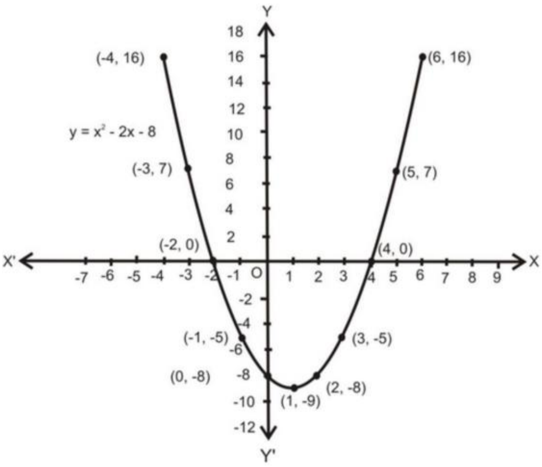
Linear Polynomial

- Graph is a straight line
- Cuts x-axis at exactly one point
- Has one zero



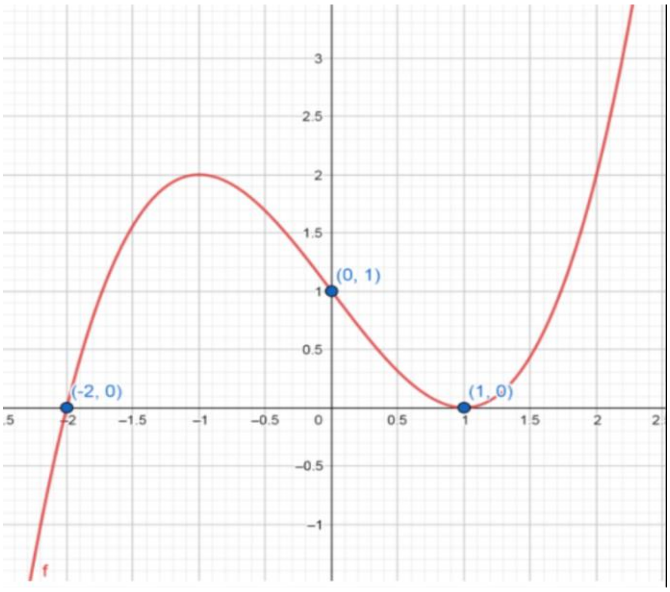
Quadratic Polynomial

- Graph is a parabola
- Can have:
    - Two zeroes (cuts x-axis at two points)
    - One zero (touches x-axis)
    - No zero (does not intersect x-axis)



Cubic Polynomial

- Graph can intersect x-axis at up to three points
- Maximum 3 zeroes



Important Result

A polynomial of degree n has **at most n zeroes**

Factorisation of Quadratic Polynomial

Quadratic polynomials can be factorised by splitting the middle term.

Example:

$x^2 – 7x + 10$

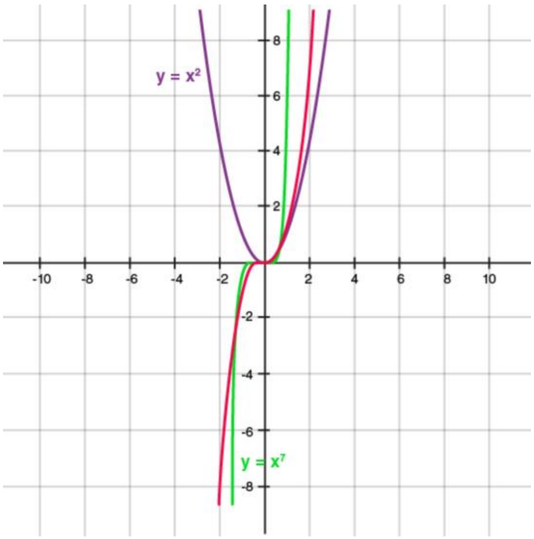
Split –7x into –5x and –2x

$x^2 – 5x – 2x + 10$
$= x(x – 5) –2(x – 5)$
$= (x – 5)(x – 2)$

Zeroes: 5 and 2

Graph of x^n

If the power (n) is even, the graph is symmetric and U-shaped.
If the power (n) is odd, the graph passes through opposite quadrants.
The graph of $–x^n$ is the reflection of x^n across the x-axis.



Relationship Between Zeroes and Coefficients

For a quadratic polynomial:

$ax^2 + bx + c$

If zeroes are α and β, then:

Sum of zeroes = $\alpha + \beta = -b/a$
Product of zeroes = $\alpha\beta = c/a$

Example:

Given polynomial: $x^2 + 6x + 8$

a = 1, b = 6, c = 8

Sum = –6/1 = –6
Product = 8/1 = 8

Finding Polynomial from Zeroes

If sum and product of zeroes are given:

Polynomial = $x^2 - (\text{sum})x + \text{product}$

Example:

Sum = 4, Product = 3

Polynomial = $x^2 - 4x + 3$

Cubic Polynomial Relations

For $ax^3 + bx^2 + cx + d$

If zeroes are α, β, γ:

- $\alpha + \beta + \gamma = -b/a$
- $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$
- $\alpha\beta\gamma = -d/a$

Division Algorithm (Polynomial Division)

If a polynomial P(x) is divided by G(x), then:

$P(x) = G(x) \times Q(x) + R(x)$

Where:

- P(x) = Dividend
- G(x) = Divisor
- Q(x) = Quotient
- R(x) = Remainder

Algebraic Identities

1. $(a+b)^2 = a^2+2ab+b^2$
2. $(a-b)^2 = a^2-2ab+b^2$
3. $(x+a)(x+b) = x^2+(a+b)x+ab$
4. $a^2-b^2 = (a+b)(a-b)$
5. $a^3-b^3 = (a-b)(a^2+ab+b^2)$
6. $a^3+b^3 = (a+b)(a^2-ab+b^2)$
7. $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
8. $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$