CHAPTER 14: PROBABILITY

1. Introduction to Probability

Probability is a branch of mathematics that deals with measuring the likelihood of occurrence of an event. It helps us quantify uncertainty in a structured way.

The value of probability always lies between 0 and 1.

- Probability = 0 → Impossible event
- Probability = 1 → Certain event

Probability can also be expressed in percentage form:

- 0% → Impossible
- 100% → Certain

Example:
The probability of getting a head in a fair coin toss is 1/2.

2. Basic Terminology

Random Experiment

A **random experiment** is an experiment whose outcome cannot be predicted with certainty in advance.

Examples:

- Tossing a coin
- Rolling a die
- Drawing a card from a deck

Outcome

An **outcome** is a possible result of a random experiment.

Example:
When a die is rolled, getting 4 is one outcome.

Event

An **event** is a set of one or more outcomes.

Example:
Getting an even number when a die is rolled → {2, 4, 6}

Sample Space

The **sample space** is the set of all possible outcomes of an experiment.

Examples:

- Coin toss → S = {H, T}
- Die roll → S = {1, 2, 3, 4, 5, 6}

3. Equally Likely Outcomes

Outcomes are equally likely if each outcome has the same probability of occurring.

Examples:

- Tossing a fair coin → Head and Tail are equally likely
- Throwing a fair die → Each number has equal chance

Not all experiments have equally likely outcomes.

Example:
If a bag contains 4 red balls and 1 blue ball, the probability of drawing a red ball is higher than that of drawing a blue ball.

4. Types of Probability

Experimental Probability

Experimental probability is based on actual observations or repeated trials.

Formula:

P(E) = Number of times event occurs / Total number of trials

Example:
If 6 out of 15 balls sold are red, then

P(Red) = 6/15 = 2/5

Theoretical Probability

Theoretical probability is calculated using reasoning and assumptions (mainly equally likely outcomes).

Formula:

P(E) = Number of favourable outcomes / Total number of possible outcomes

Example:
Probability of getting a head in a coin toss:

P(H) = 1/2

Elementary Event

An event that consists of only one outcome is called an elementary event.

Example:
Getting “3” when a die is thrown.

Complementary Events

For any event E, the event “not E” is called its **complement.**

Relationship:

P(E) + P(E') = 1

P(E') = 1 – P(E)

Example:

If P(Head) = 1/2, then

P(Not Head) = 1 – 1/2 = 1/2

Range of Probability

For any event E:

$$0 \leq P(E) \leq 1$$

This means probability can never be negative or greater than 1.

Impossible Event

An event that cannot occur has probability 0.

Example:
Getting 7 when a die is thrown.

P(E) = 0

Sure Event

An event that always occurs has probability 1.

Example:
Getting a number less than 7 when a die is thrown.

P(E) = 1

Sum of Probabilities

The sum of probabilities of all elementary events of an experiment is always equal to 1.

Example:
For a coin toss:

P(H) + P(T) = 1/2 + 1/2 = 1

Probability with Coins

11.1 Single Coin

Sample space: {H, T}

P(H) = 1/2
P(T) = 1/2

11.2 Two Coins

Sample space:
(HH), (HT), (TH), (TT)

Total outcomes = 4

Example:
Probability of getting at least one head:

Favourable outcomes = (HH), (HT), (TH) → 3

P = 3/4

Probability with Dice

12.1 Single Die

Sample space = {1, 2, 3, 4, 5, 6}

Example:
Probability of getting a number greater than 4:

Favourable outcomes = {5, 6} → 2

P = 2/6 = 1/3

12.2 Two Dice

Total outcomes = 6 × 6 = 36

Example:
Probability that sum is 8:

Favourable outcomes:
(2,6), (3,5), (4,4), (5,3), (6,2) → 5

P = 5/36

Probability with Playing Cards

A standard deck contains 52 cards divided into 4 suits:

- Hearts and Diamonds (Red)
- Spades and Clubs (Black)

Each suit has 13 cards.

Face cards: King, Queen, Jack

TYPE	NUMBER
TOTAL CARDS	52
RED	26
BLACK	26
FACE CARDS	12
ACES	4

Example:
Probability of drawing an ace:

Number of aces = 4

P = 4/52 = 1/13

Probability with Objects (Balls, Marbles, etc.)

Example:

A box contains 3 blue, 2 white and 4 red marbles.

Total outcomes = 9

P(White) = 2/9

P(Blue) = 3/9 = 1/3

P(Red) = 4/9

Complement Rule Applications

If finding direct probability is difficult, use complement:

P(E) = 1 – P(Not E)

Example:
Probability of at least one head in two coin tosses:

P(No head) = 1/4

P(At least one head) = 1 – 1/4 = 3/4

Real-Life Example

Probability that two people have the same birthday:

P(Same) = 1/365

Probability that they have different birthdays:

P(Different) = 364/365

Geometric Probability (Basic Idea)

Used when outcomes are infinite.

Formula:

Probability = Favourable area / Total area

Example:
If a point is randomly chosen in a region, probability depends on area ratio.

Important Results Summary

1. P(E) = Favourable outcomes / Total outcomes
2. $0 \leq P(E) \leq 1$
3. P(Impossible event) = 0
4. P(Certain event) = 1
5. P(E) + P(E') = 1

6. Sum of all probabilities = 1