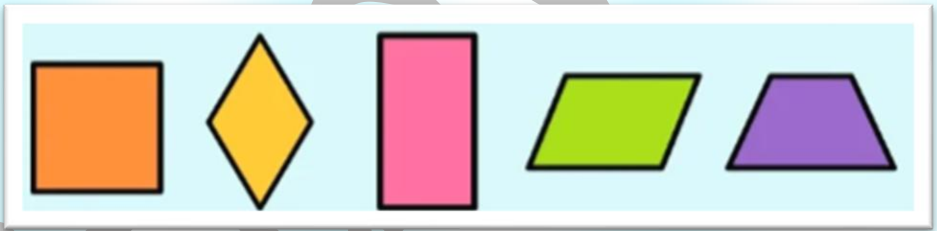
CHAPTER 8: QUADRILATERALS

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

1. Introduction to Quadrilaterals

A quadrilateral is a polygon with four sides and four vertices. The sum of the interior angles of any quadrilateral is 360°. Types of quadrilaterals: Parallelogram, Rectangle, Square, Rhombus, Trapezium, Kite.



2. Angle Sum Property of Quadrilateral

Proof: Divide quadrilateral into two triangles; each triangle sums to 180°, so quadrilateral angles sum to 360°.

Example: Angles in ratio 3:5:9:13

Let x be the common ratio factor, then

$3x + 5x + 9x + 13x = 360 \Rightarrow 30x = 360 \Rightarrow x = 12$

Angles are 36°, 60°, 108°, and 156°.

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3. Types of Quadrilaterals & Their Properties

Quadrilateral	Definition	Key Properties
Parallelogram	Both pairs of opposite sides parallel	Opposite sides equal; opposite angles equal; diagonals bisect each other; consecutive angles supplementary
Rectangle	Parallelogram with all angles 90°	Diagonals equal; opposite sides equal
Square	Rectangle + Rhombus	All sides equal; all angles 90°; diagonals equal and bisect at right angles
Rhombus	All sides equal	Diagonals bisect at right angles; opposite sides parallel
Trapezium	One pair opposite sides parallel	Non-parallel sides unequal generally; angles adjacent to parallel sides supplementary
Kite	Two pairs adjacent equal sides	One pair opposite angles equal; one diagonal bisects other

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4. Properties of Parallelograms

- **Opposite sides equal and parallel.**
- **Opposite angles equal.**

- Diagonals bisect each other.
- Consecutive angles are supplementary.

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5. Criteria to Prove a Quadrilateral is a Parallelogram

A quadrilateral $ABCD$ is a parallelogram if:

- Both pairs of opposite sides are equal.
- Both pairs of opposite sides are parallel.
- One pair of opposite sides is both equal and parallel.
- Diagonals bisect each other.
  Each condition comes with stepwise proof and examples.

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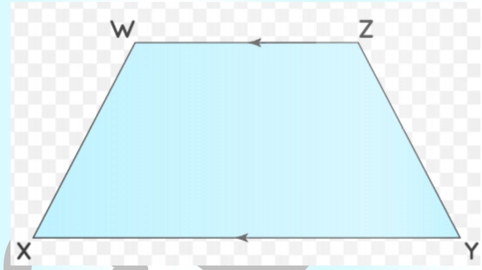
6. Midpoint Theorem and Applications

- Statement: The segment joining midpoints of two sides of a triangle is parallel to the third side and half its length.
- Proof with diagram.
- Application in quadrilaterals: If midpoints of all sides of any quadrilateral are joined, they form a parallelogram.
- Problems solved step-by-step.

7. Types of Quadrilaterals Other Than Parallelogram

1. Trapezium (Trapezoid)



- **Definition:** A trapezium is a quadrilateral with exactly one pair of opposite sides parallel. The parallel sides are called bases, and the non-parallel sides are called legs.
- **Properties:**
  - One pair of opposite sides are parallel.
  - The sum of the adjacent angles between a leg and the bases is 180° (supplementary).
  - The diagonals intersect but do not necessarily bisect each other.
  - In an isosceles trapezium, the legs (non-parallel sides) are equal in length, and the base angles are equal.
- **Area of trapezium formula:**

$$\text{Area} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

- **Example:** A trapezium has parallel sides of 8 cm and 5 cm, and height 4 cm. Find its area.

$$\frac{1}{2} \times (8+5) \times 4 = \frac{1}{2} \times 13 \times 4 = 26\ \text{cm}^2$$

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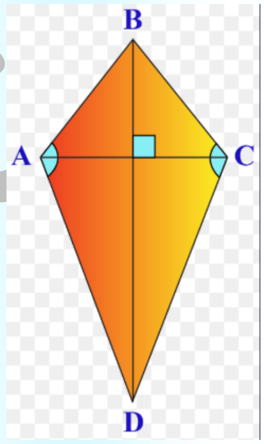
2. Kite



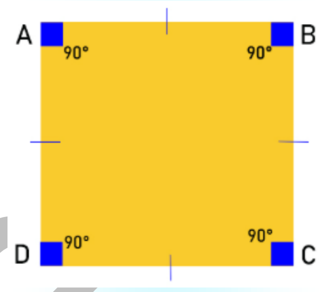
- **Definition: A kite is a quadrilateral with two pairs of adjacent sides equal. Unlike parallelograms, the equal sides are next to each other, not opposite.**
- **Properties:**
  - **Two pairs of equal adjacent sides.**
  - **One pair of opposite angles (the angles between unequal sides) are equal.**
  - **The diagonals intersect at right angles (are perpendicular).**
  - **The longer diagonal bisects the shorter diagonal.**
  - **The kite is symmetric about the longer diagonal.**
- **Area of kite formula:**

$$\text{Area} = \frac{1}{2} \times \text{diagonal}_1 \times \text{diagonal}_2$$

- **Example: In a kite, diagonals are 12 cm and 8 cm. Find the area.**

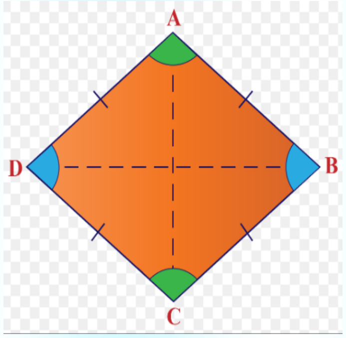
$$\frac{1}{2} \times 12 \times 8 = 48\ \text{cm}^2$$

3. Rectangle



- A rectangle is a parallelogram with all angles equal to 90°.
- Opposite sides are equal and parallel, diagonals are equal and bisect each other.
- Area = length × breadth.

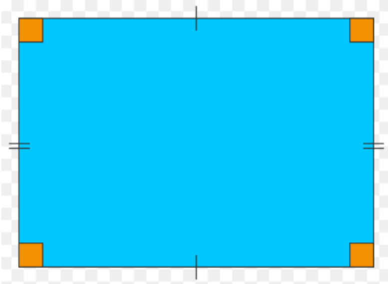
4. Square



- A square is a rectangle with all sides equal.
- All angles 90°, diagonals equal and bisect each other at right angles.
- Area = side^2.

5. Rhombus

- A rhombus is a parallelogram with all sides equal.
- Opposite angles equal, diagonals bisect each other at right angles but are generally not equal.
- Area = $\frac{1}{2} \times$ (product of diagonals).



8. Summary Table of Quadrilaterals

To help you quickly review and compare the key features, here is a summary table that consolidates the essential properties of different types of quadrilaterals. Use this as a handy reference to remember their characteristics and distinctions clearly.

Quadrilateral	Sides	Angles	Diagonals
Parallelogram	Opposite sides equal and parallel	Opposite angles equal	Bisect each other
Rectangle	Opposite sides equal and parallel	All 90°	Equal and bisect
Square	All sides equal and parallel	All 90°	Equal and bisect at right angles
Rhombus	All sides equal	Opposite angles equal	Bisect at right angles
Trapezium	One pair of sides parallel	Adjacent angles on legs supplementary	Intersect but do not bisect
Kite	Two pairs of adjacent sides equal	One pair of opposite angles equal	Diagonals perpendicular, one bisects the other

9. Solved Questions with Detailed Explanations

Q1: In parallelogram ABCD, prove that AB = CD and BC = AD.

Sol: Given: ABCD is a parallelogram.

To prove: AB = CD and BC = AD.

Proof:

1. In parallelogram ABCD, opposite sides are parallel: AB ∥ CD and AD ∥ BC. (Definition)

2. Let the diagonals AC and BD intersect at O.

3. In triangles ΔAOB and ΔCOD:

- ∠OAB = ∠OCD (alternate interior angles, since AB ∥ CD)

- ∠OBA = ∠ODC (alternate interior angles, since AD ∥ BC)

- AO = OC and BO = OD (diagonals of a parallelogram bisect each other)

4. Therefore ΔAOB ≅ ΔCOD by SAS congruence.

5. From congruence corresponding sides, AB = CD and BC = AD.

Hence AB = CD and BC = AD.

Q2: In rectangle ABCD, prove that AC = BD.

Sol: Given: ABCD is a rectangle.

To prove: AC = BD.

Proof:

1. In a rectangle opposite sides are equal and all angles are 90°. So AB = CD and BC = AD.

2. Consider triangles ΔABC and ΔCDA.

- AB = CD (opposite sides)

- BC = DA (opposite sides)

- ∠ABC = ∠CDA = 90° (right angles)

3. So ΔABC ≅ ΔCDA by SAS (two sides and included angle).

4. From congruence corresponding sides, AC = BD.

Hence AC = BD.

Q3: In quadrilateral ABCD, diagonals AC and BD meet at O. Given AO = OC and BO = OD. Prove that ABCD is a parallelogram.

Sol: Given: In quadrilateral ABCD, AO = OC and BO = OD.

To prove: ABCD is a parallelogram.

Proof:

1. In triangles ΔAOB and ΔCOD:

- AO = OC (given)

- BO = OD (given)

- ∠AOB = ∠COD (vertically opposite angles)

2. By SAS, ΔAOB ≅ ΔCOD.

3. Corresponding angles give ∠OAB = ∠OCD and ∠OBA = ∠ODC.

These are alternate interior angles, so AB ∥ CD and AD ∥ BC.

4. Since both pairs of opposite sides are parallel, ABCD is a parallelogram.

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10. Practice Drill Questions with Solutions

1. In a kite ABCD, AB = AD and CB = CD. If ∠BAD = 50°, find ∠BCD.

2. ABCD is a trapezium in which AB ∥ CD. If ∠A = 70° and ∠D = 110°, find ∠B and ∠C.

3. In a rhombus PQRS, diagonal PR = 12 cm and diagonal QS = 16 cm. Find the length of each side.

4. In a rectangle ABCD, diagonal AC makes an angle of 40° with side AB. Find ∠DAC and ∠ACB.

5. In an isosceles trapezium ABCD, AB ∥ CD and AD = BC. If ∠A = 75°, find ∠B, ∠C, and ∠D.

6. In quadrilateral ABCD, sides AB = BC = CD. What special type of quadrilateral is this? Justify by examining angles or diagonals.

7. In square PQRS, a point T is taken on diagonal PR such that PT = TR. Prove that triangle PST is congruent to triangle RST.

8. ABCD is a general quadrilateral (not parallelogram). Diagonals AC and BD intersect at O. If AO/OC = 2 and BO/OD = 3, show whether ABCD can be a parallelogram or not.

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11. Common Errors & How to Avoid Them

- Assuming diagonals always bisect at right angles.
- Confusing properties of rhombus and square.
- Incorrect angle calculations ignoring supplementary angles.
- Omitting reasons in proofs.
- Tips to carefully analyze given quadrilaterals before applying properties.

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