CHAPTER 1: REAL NUMBERS

Introduction

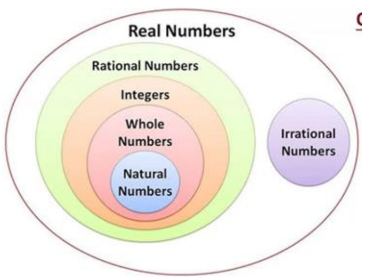
Real numbers form the foundation of mathematics and are used in almost every calculation in daily life. These include counting numbers, integers, fractions, decimals, and irrational numbers. In this chapter, we study important concepts such as Euclid's division algorithm, the Fundamental Theorem of Arithmetic, methods of finding HCF and LCM, and properties of irrational numbers.

Real Numbers

Real numbers include all numbers that can be represented on a number line.

Examples:

- Integers: -5, 0, 7
- Fractions: 3/4, -2/3
- Decimals: 1.25, 0.666...
- Irrational numbers: √2, √5

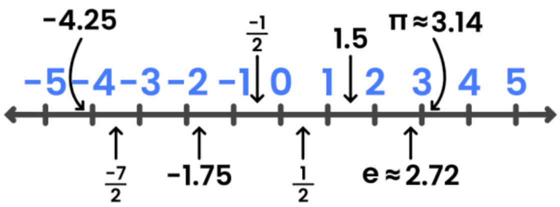


Key Properties:

- Real numbers consist of both rational and irrational numbers.
- Every real number has a unique position on the number line.

Classification of Real Numbers

Category	Description	Examples
Natural Numbers	Counting numbers	1, 2, 3
Whole Numbers	Natural numbers + 0	0, 1, 2
Integers	Positive and negative numbers	-3, 0, 5
Rational Numbers	Numbers in the form p/q	2/3, -5/7
Irrational Numbers	Cannot be expressed as p/q	√3, π



Euclid's Division Lemma

Statement:

For any two integers a and b (b ≠ 0), there exist unique integers q and r such that:

$a = bq + r$, where $0 \le r < b$

Explanation:

This means when a number is divided by another, the result always gives a quotient and a remainder that is less than the divisor.

Key Points:

- The remainder is **always less than the divisor**
- If remainder ≥ divisor → calculation is incorrect

Euclid's Division Algorithm (for HCF)

This algorithm is used to find the Highest Common Factor (HCF) of two numbers.

Steps:

1. Divide the larger number by the smaller number.
2. Replace the numbers with divisor and remainder.
3. Repeat the process until remainder becomes zero.
4. The last non-zero divisor is the HCF.

Example:

Find HCF of 72 and 30
$72 = 30 \times 2 + 12$
$30 = 12 \times 2 + 6$
$12 = 6 \times 2 + 0$

HCF = 6

Prime Factorisation

Prime factorisation is the process of expressing a number as a product of prime numbers.

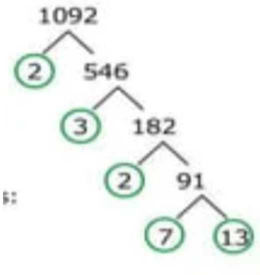
Example:

$60 = 2^2 \times 3 \times 5$

Fundamental Theorem of Arithmetic

Statement:

Every **composite number** can be expressed as a product of prime numbers, and this representation is unique except for the order of factors.



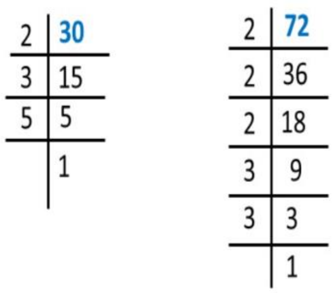
Example:

$1092 = 2 \times 3 \times 2 \times 7 \times 13$
Rearranging factors does not change the prime composition.

Finding HCF by Prime Factorisation

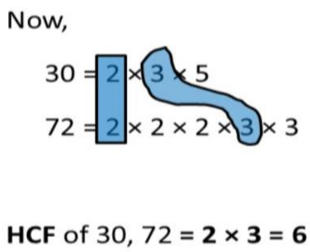
Method:

- Express numbers as product of prime factors.
- Select common prime factors with smallest powers.



Example:

Find HCF of 24 and 36



Fast Method to Find HCF

Use prime factorisation quickly:

Example:
HCF(24, 36)

24 = $2^3 \times 3$
36 = $2^2 \times 3^2$

Take common primes with **smallest powers**
HCF = $2^2 \times 3 = 12$

Finding LCM by Prime Factorisation

Method:

- Express numbers as product of primes.
- Take all prime factors with highest powers.

Example:

Find LCM of 18 and 30
18 = 2×3^2
30 = $2 \times 3 \times 5$

LCM = $2 \times 3^2 \times 5 = 90$

Relationship Between HCF and LCM

For any two positive integers a and b:

a × b = HCF × LCM

Example:

For 18 and 30
18 × 30 = 540
HCF = 6, LCM = 90
6 × 90 = 540

This relation does not hold for more than two numbers.

Formula:

$$LCM = \frac{\text{Product of numbers}}{\text{HCF}}$$

Applications of HCF and LCM

- Finding common meeting times
- Synchronising repeated events
- Solving problems related to cycles and intervals

Irrational Numbers

Irrational Numbers, that cannot be expressed in the form p/q, where p and q are integers and $q \neq 0$.

Examples:

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π

Important Theorem

If a prime number divides the square of a number, then it divides the number itself.

If a prime number divides a^2, then it must divide a

Proof by Contradiction

This method is used to prove statements by assuming the opposite and arriving at a contradiction.

Proof that $\sqrt{2}$ is Irrational

1. Assume $\sqrt{2} = a/b$, where a and b are coprime.
2. Squaring both sides: $2b^2 = a^2$
3. Therefore, a is divisible by 2.
4. Let $a = 2c$
5. Substituting: $b^2 = 2c^2$
6. Hence, b is also divisible by 2.

This contradicts the assumption that a and b are coprime.
Therefore, $\sqrt{2}$ is irrational.

Properties of Irrational Numbers

- Sum or difference of a rational and irrational number is irrational.
- Product of a non-zero rational and irrational number is irrational.

Rational Numbers and Decimal Expansion

Rational Numbers: Numbers that can be written as p/q.

Types of Decimal Expansion

Terminating Decimal:

Decimal ends after finite digits
Example: 0.25, 1.5

Non-Terminating Decimal:

Decimal does not end

Types:

- Recurring (Repeating): 0.333...
- Non-recurring: π

```
p/q → Simplify
     ↓
Denominator only 2,5?
     ↓
YES → Terminating
NO → Non-terminating
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TRICK

1. Simplify the fraction (HCF = 1)
2. Check denominator

Denominator	Result
Only 2 and/or 5	Terminating
Any other prime factor	Non-terminating

Examples:

- 7/40 → Terminating ☑
- 7/12 → Non-terminating ✕

Condition for Terminating Decimal

A rational number p/q has a terminating decimal if:

1. HCF(p, q) = 1
2. Denominator has only prime factors 2 and/or 5

Examples:

- 3/40 → Terminating (40 = $2^3 \times 5$)
- 7/12 → Non-terminating (12 includes factor 3)

Common Mistakes to Avoid ✕

- Taking highest power in HCF (wrong)
- Taking smallest power in LCM (wrong)

- Not simplifying fractions
- Wrong identification of decimal type

Quick Revision Formula Box 📦

- $a = bq + r$
- HCF × LCM = Product
- Terminating condition → denominator = $2^n \times 5^m$
- √(non-perfect square) = irrational

Important Question Types

Frequently asked:

- Steps of Euclid's Algorithm
- Finding HCF & LCM
- Checking terminating/non-terminating decimals
- Proof that $\sqrt{2}$ is irrational
- Word problems (bells, traffic signals, cycles)