# CHAPTER 11: SURFACE AREAS AND VOLUMES

### YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

## 1. Fundamental Concepts

**Three-dimensional solids extend 2D shapes into depth, occupying real space unlike flat figures.**

**Surface Area: Measures outer covering needed for painting or wrapping.**

**Lateral Surface Area (LSA): Sides only, excluding top/bottom (e.g., cardboard for cylinder sides).**

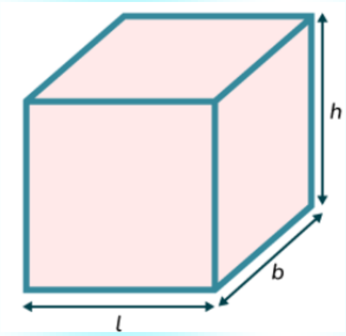
**Total Surface Area (TSA): LSA + all bases (complete wrapping).**

**Volume: Internal space for filling (e.g., water capacity), always cubic units.**

**Visual Aid Insight: Imagine nets—unfold a cylinder to 2 circles + rectangle; this reveals why LSA = circumference × height.**

**Units: Surface area in $m^2/cm^2$; volume in $m^3/cm^3$. Convert consistently (1 m = 100 cm).**

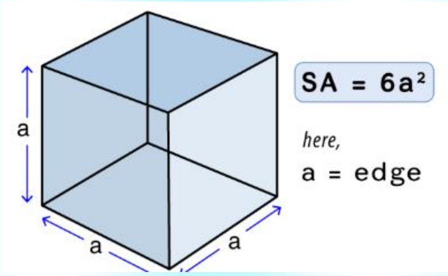
## 2. Cuboid and Cube




**Cuboid: Rectangular box with 3 pairs of identical faces (length $l$, breadth $b$, height $h$).**

**Conceptual Derivation:**

- **Faces: 2 of $lb$ (top/bottom), 2 of $bh$ (front/back), 2 of $hl$ (sides).**
- **TSA = $2(lb + bh + hl)$—sum all exposed surfaces.**
- **LSA = $2h(l + b)$—vertical sides only.**
- **Volume = base area × height = $l \times b \times h$.**



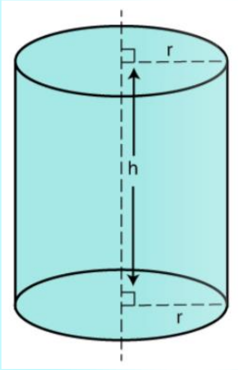

**Cube: Special cuboid where $l = b = h = a$.**

- **TSA = $6a^2$, LSA = $4a^2$, Volume = $a^3$.**
- **Why Cube Scales Differently: Doubling edge quadruples SA but octuples volume—key for material efficiency in packaging.**

**Real-Life: Room painting (TSA minus floor/ceiling), dice volume.**



## 3. Right Circular Cylinder




**Concept: Circular bases joined by curved surface; like a tin can.**

**Derivation:**

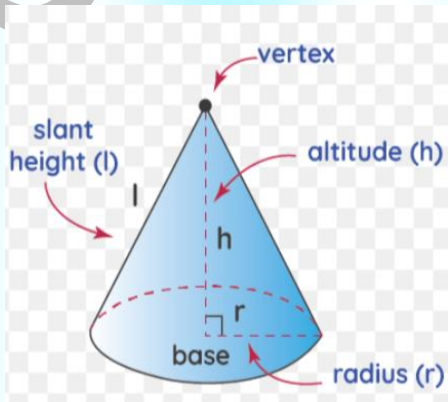
- **Base circumference = $2\pi r$.**

- Unroll side: rectangle of length $2\pi r$, width $h$ → LSA = $2\pi rh$.
- TSA = LSA + 2 bases = $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$.
- Volume = base area × height = $\pi r^2 h$.

**Key Insight:** LSA unchanged if rotated—uniform curvature. Use for open-top tanks (TSA = LSA + 1 base).

## 4. Right Circular Cone

**Concept:** Circular base tapering to apex; ice cream cone shape.

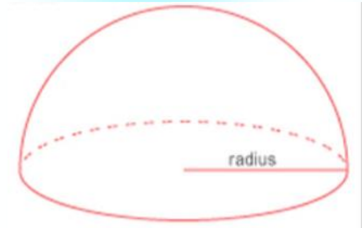



**Derivation:**

- Slant height $l = \sqrt{r^2 + h^2}$ (Pythagoras on axial triangle).
- Unroll curved surface: sector with arc $2\pi r$, radius $l$ → CSA = $\pi rl$.
- TSA = CSA + base = $\pi r(l + r)$.
- Volume = $\frac{1}{3}\pi r^2 h$ (1/3 cylinder volume due to triangular cross-section).

**Visualization:** Slice vertically—isosceles triangle rotated around height.

## 5. Sphere and Hemisphere




**Sphere: Perfectly round; no edges or faces.**

**Derivation Insight: Surface approximated by infinite rings; TSA = $4\pi r^2$. Volume via integration or Cavalieri's principle = $\frac{4}{3}\pi r^3$.**

**Hemisphere: Half-sphere + flat base.**

- **CSA = curved half = $2\pi r^2$.**
- **TSA = CSA + base = $3\pi r^2$.**
- **Volume = $\frac{2}{3}\pi r^3$.**

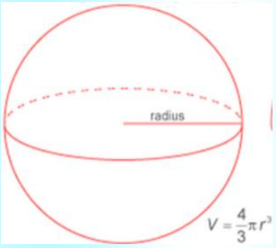
**Application: Globes (TSA minus poles), bowls (hemisphere volume).**

## 6. Composite Solids

**Concept: Real objects combine shapes (e.g., tent = cone + cylinder). Calculate by addition/subtraction, ignoring hidden surfaces.**

**Strategy:**

1. **Sketch and label parts.**
2. **TSA: Exposed surfaces only (subtract overlaps).**
3. **Volume: Sum internal spaces.**




**Example Breakdown: Cuboidal room with hemispherical dome.**

- **TSA: Cuboid TSA - roof area + hemisphere TSA.**
- **Visualize: Dome base merges with roof—no double base.**

## 7. Complete Formula Reference

| Solid | LSA/CSA | TSA | Volume |
|---|---|---|---|
| Cuboid | $2h(l+b)$ | $2(lb+bh+hl)$ | $lbh$ |
| Cube | $4a^2$ | $6a^2$ | $a^3$ |
| Cylinder | $2\pi rh$ | $2\pi r(r+h)$ | $\pi r^2h$ |
| Cone | $\pi rl$ | $\pi r(l+r)$ | $\frac{1}{3}\pi r^2h$ |
| Sphere | - | $4\pi r^2$ | $\frac{4}{3}\pi r^3$ |
| Hemisphere | $2\pi r^2$ | $3\pi r^2$ | $\frac{2}{3}\pi r^3$ |

## 8. Solved Examples with Step-by-Step Reasoning

**Ex 1 (Cuboid): $l = 15$ cm, $b = 10$ cm, $h = 6$ cm. Find TSA, volume.**

- **Pairs: $lb = 150$ (×2=300), $bh = 60$ (×2=120), $hl = 90$ (×2=180).**
- **TSA = 300+120+180 = 600 cm².**
- **Volume = 15×10×6 = 900 cm³. *Reason: Multiplies dimensions for space.***

**Ex 2 (Cylinder, Open Top): $r = 7$ cm, $h = 10$ cm.**

- **TSA = CSA + 1 base = $2\pi \times 7 \times 10 + \pi \times 49 = 440\pi + 49\pi = 489\pi$ cm². *Hidden: No top base.***

**Ex 3 (Cone): $r = 4$ cm, $h = 3$ cm.**

- **$l = \sqrt{16+9} = 5$ cm.**

- Volume = $\frac{1}{3}\pi \times 16 \times 3 = 16\pi$ cm³. *1/3 factor from averaging cross-sections.*

**Ex 4 (Composite): Cone (r=5 cm, h=12 cm) + hemisphere (r=5 cm).**

- $l = \sqrt{25 + 144} = 13$ cm.
- TSA = cone CSA + hemisphere TSA = $\pi \times 5 \times 13 + 3\pi \times 25 = 65\pi + 75\pi = 140\pi$ cm² (base hidden).
- Volume = $\frac{1}{3}\pi \times 25 \times 12 + \frac{2}{3}\pi \times 125 = 100\pi + \frac{250}{3}\pi$. *Add capacities.*

**Ex 5 (Advanced): Sphere melted into cone (same r,h). Volumes equal → $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$ → $h = 4r$.** *Proves shape transformation conservation.*

## 9. Common Errors and Fixes

- Error: Wrong slant height → *Fix*: Always Pythagoras.
- Error: Double bases in composites → *Fix*: Visualize hidden parts.
- Error: Unit mismatch → *Fix*: Convert pre-calculation.

## 10.Practice Mastery Drills

**1. A cylinder and a cone have the same base radius and height. If the curved surface area of the cylinder is 154 cm², find the curved surface area of the cone.**

2. The radius of a sphere is doubled. By what factor does its volume increase?

3. A solid metal cube of edge 6 cm is melted and recast into a sphere. Find the radius of the sphere.

4. A cone has height 12 cm and slant height 13 cm. Find its total surface area if its radius is doubled.

5. A cylindrical pipe has outer radius 7 cm and inner radius 5 cm. Find the volume of material used in 20 cm length of the pipe.