CHAPTER 12: SURFACE AREAS AND VOLUMES

12.1 Introduction

In our daily life, we see many 3D objects like boxes, tanks, bottles, cones, and balls. These objects are not always simple shapes — many are **combinations of basic solids**.

Basic solids you already know:

- Cube
- Cuboid
- Cylinder
- Cone
- Sphere
- Hemisphere

But real-life objects are often made by combining these shapes, for example:

- Water tank → Cylinder + hemispheres
- Test tube → Cylinder + hemisphere
- Toy rocket → Cylinder + cone

In this chapter, we learn how to:

- Find **surface areas** of combined solids
- Calculate **volumes** of such objects

12.2 Types of Surface Areas

Before moving to combinations, recall:

1. Curved Surface Area (CSA)

Only curved part (no base)

2. Lateral Surface Area (LSA)

Side surfaces only (excluding top & bottom)

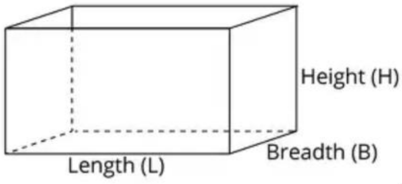
3. Total Surface Area (TSA)

Entire outer surface (all faces)

12.3 Important Formulas

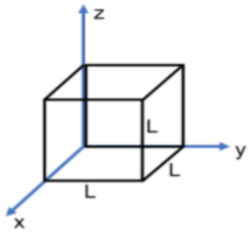
1. Cuboid

- TSA = $2(lb + bh + lh)$
- LSA = $2h(l + b)$
- Volume = $l \times b \times h$



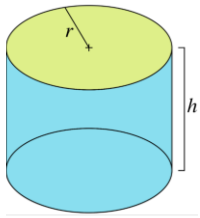
2. Cube

- TSA = $6l^2$
- LSA = $4l^2$
- Volume = l^3



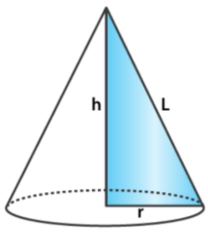
3. Cylinder

- CSA = $2\pi rh$
- TSA = $2\pi r(h + r)$
- Volume = $\pi r^2 h$



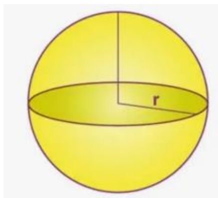
4. Cone

- CSA = πrl
- TSA = $\pi r(l + r)$
- Volume = $(1/3)\pi r^2 h$



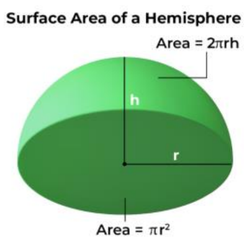
5. Sphere

- Surface Area = $4\pi r^2$
- Volume = $(4/3)\pi r^3$



6. Hemisphere

- CSA = $2\pi r^2$
- TSA = $3\pi r^2$
- Volume = $(2/3)\pi r^3$



Surface Area of Combination of Solids

Key Concept

When solids are joined:

- **Common surfaces are not counted**
- Only **visible surfaces** are included

Case 1: Cylinder + Two Hemispheres

Example: Capsule / Tank

Surface area =
CSA of cylinder + CSA of both hemispheres

$= 2\pi rh + 2(2\pi r^2)$

Case 2: Cone + Hemisphere

Surface area =
CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2$

Case 3: Cube + Hemisphere

Surface area =
TSA of cube – area covered by hemisphere + CSA of hemisphere

Important Tip

Always:

1. Identify hidden surfaces
2. Subtract overlapped areas
3. Add only visible parts

Solved Concept (Understanding Approach)

Example Type 1: Toy Shape

Cone on hemisphere

Steps:

1. Find radius
2. Find slant height of cone
3. Apply:

- CSA cone = πrl
- CSA hemisphere = $2\pi r^2$

4. Add both

Example Type 2: Cube with Hemisphere

Steps:

1. TSA of cube = $6l^2$
2. Subtract area where hemisphere sits = πr^2
3. Add curved area of hemisphere = $2\pi r^2$

Final:
TSA = $6l^2 + \pi r^2$

Example Type 3: Cylinder with Hollow Part

Steps:

- Outer surface only considered
- Internal removed parts ignored

Volume of Combination of Solids

Key Concept

Unlike surface area:
Volumes are always added (or subtracted if hollow)

Case 1: Combined Solid

Volume = Volume□ + Volume□

Case 2: Hollow Object

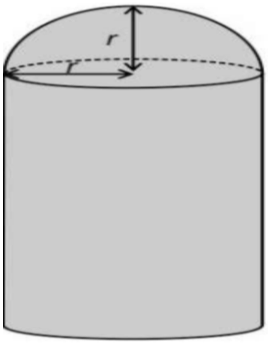
Volume = Outer volume – Inner volume

Case 3: Converted Shape

Volume remains SAME

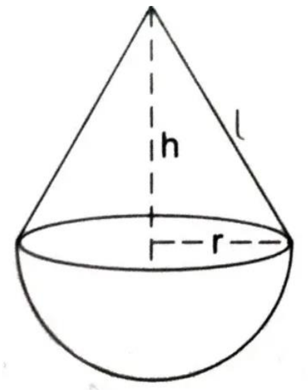
Important Volume Cases

1. Cylinder + Hemisphere



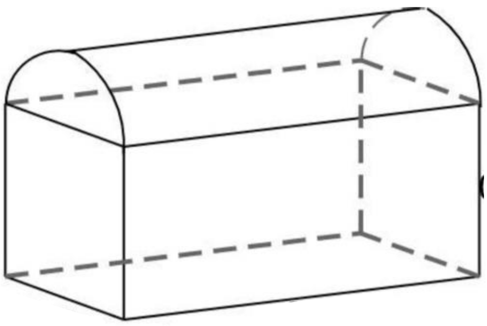
$V = \pi r^2 h + (2/3)\pi r^3$

2. Cone + Hemisphere



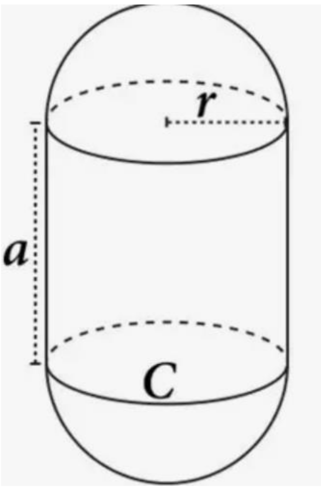
$V = (1/3)\pi r^2 h + (2/3)\pi r^3$

3. Cuboid + Half Cylinder



$+ (1/2)\pi r^2 h$

4. Capsule Shape



V = Cylinder + 2 hemispheres

Capacity vs Volume

- **Volume** → Total space
- **Capacity** → Usable space

Example:
Glass with raised bottom → capacity < volume

Step-by-Step Problem Strategy

🔷 **For Surface Area:**

1. Break into known shapes
2. Identify hidden parts
3. Use CSA/TSA carefully
4. Add visible areas

🔷 **For Volume:**

1. Divide into shapes
2. Use formulas separately
3. Add or subtract volumes

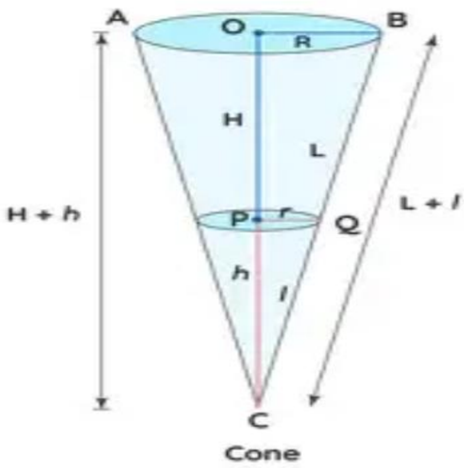
Real-Life Applications

- Water tanks
- Capsules (medicine)

- Ice-cream cones
- Storage containers
- Architecture designs

Special Case: Frustum of Cone

When top of cone is cut:



Formulas:

- CSA = π(r□ + r□)l
- TSA = π(r□ + r□)l + π(r□² + r□²)
- Volume = (1/3)πh(r□² + r□² + r□r□)

Conversion of Solids

When one shape is melted and reshaped:

Volume remains constant

Example:
Sphere → Cylinder

Equation:
Volume of sphere = Volume of cylinder