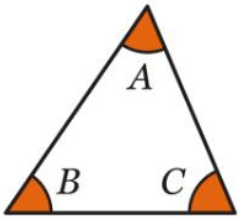
CHAPTER 7: TRIANGLES

YOUR ULTIMATE GUIDE TO MASTERING THE FOUNDATIONS OF MATHEMATICS

1. Triangle Anatomy

- **Definition: A triangle is a closed figure with three straight sides (AB, BC, CA), three angles (∠A, ∠B, ∠C), and three vertices (A, B, C).**
- **Key Point: The sum of all angles in a triangle is always 180°.**



$$A + B + C = 180°$$

2. Types of Triangles

2.1 By Sides:

- **Equilateral: All sides equal; all angles 60°.**
- **Isosceles: Two sides equal; base angles equal.**
- **Scalene: All sides and angles are different.**

2.2 By Angles:

- **Acute: All angles < 90°.**
- **Right: One angle is 90°.**
- **Obtuse: One angle > 90°.**

Type	Sides	Angles
Equilateral	a = b = c	all 60°
Isosceles	a = b ≠ c	base angles equal
Scalene	all sides unequal	all angles unequal
Right-angled	one angle = 90°	two acute

Real-Life Examples

- **Traffic signs, sails of boats (isosceles), ramps (right-angled), pyramids, triangular sandwich pieces (scalene).**

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3. Congruence in Triangles

What is Congruence?

- **Congruent figures are identical in size and shape.**
- **For triangles: ΔABC ≅ ΔPQR means all angles and sides match perfectly.**

Correspondence matters! Always state which vertices match.

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4. Criteria for Congruence

- **SSS (Side-Side-Side): All sides of one triangle equal to those of another.**
- **SAS (Side-Angle-Side): Two sides and their included angle are equal.**
- **ASA (Angle-Side-Angle): Two angles and the included side are equal.**
- **AAS (Angle-Angle-Side): Two angles and any side are equal.**
- **RHS (Right angle-Hypotenuse-Side): For right triangles, hypotenuse and one other side are equal.**

Criterion	What's Equal?	Applies When
SSS	All three sides	all sides marked/matched
SAS	Two sides + angle	angle between marked sides
ASA	Two angles + side	side between marked angles
AAS	Two angles + any side	any side and two angles
RHS	Right angle, hyp., side	right triangle only

5. Important Theorems and Proofs (with Steps):

1. Isosceles Triangle Theorem

- **Statement: Angles opposite equal sides of a triangle are equal.**
- **Proof:**

- Draw bisector from vertex between equal sides to base.
- Show the resulting triangles are congruent using SAS.
- CPCT (Corresponding Parts of Congruent Triangles): base angles are equal.

2. **Converse of Isosceles Theorem**
   - If two angles are equal, sides opposite those angles are equal.
   - Use congruence and construction to prove.
3. **Triangle Inequality**
   - For any triangle, the sum of any two sides is greater than the third side.
   - Example for triangle ABC:
     - AB + BC > CA
     - BC + CA > AB
     - CA + AB > BC

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6. Essential Properties Table

Property	Statement
Angle Sum	$\angle A + \angle B + \angle C = 180°$
Triangle Inequality	Any two sides' sum > third side
Exterior Angle Property	Exterior angle = sum of two opposite interior angles

7. Step-by-Step Solutions (NCERT, Advanced)

Q1: AB = AC in ΔABC, altitude from B meets AC at D, from C at E. Show BD = CE.

Solution:

1. **Triangles ABD and ACE:**
    - **AB = AC (given)**
    - **∠ABD = ∠ACE = 90° (altitude)**
    - **AD = AE (common)**
    - **RHS congruence ⇒ ΔABD ≅ ΔACE**
    - **BD = CE (CPCT)**

Q2: In ΔXYZ, XY = YZ = 5 cm, ∠X = ∠Z. Find triangle type and all angles.

Solution:

- **Isosceles (XY = YZ).**
- **Use angle sum property, base angles theorem to solve for other angles.**

8. HOTS (Higher Order Thinking Skills)

a) Given side lengths 6 cm, 10 cm, 5 cm: Is this triangle possible?

b) Check all triangle inequalities.

c) In a right triangle, hypotenuse = 10 cm, one side = 6 cm. Find all possibilities, use diagrams.

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9. Common Errors & Error Bank

- Using invalid congruence criteria (SSA, AAA): not valid!
- Mistaking corresponding vertices when writing $\Delta ABC \cong \Delta DEF$.
- Including/excluding the wrong angle (be careful with SAS/ASA).
- Skipping triangle inequality checks before drawing any triangle.
- Ignoring correct, precise diagram labeling.

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10. Practice Drills & Exam-Style Questions

1. Draw and label all types of triangles. Color code equal sides/angles.
2. Mark congruent pairs with arcs/tick marks.
3. Prove: Bisector of vertical angle of isosceles triangle bisects base—show all steps.
4. In ΔPQR, $PQ = QR$, $\angle P = 40°$. Find the other angles with full steps.

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11. Important Questions with Stepwise Solutions

Q: Prove two triangles with two sides and the included angle equal are congruent (SAS).

Solution:

1. **List the corresponding sides and included angle.**
2. **Use SAS for congruence.**
3. **Corresponding parts (CPCT): deduce all equal sides/angles.**

Q: Given AB = AC = 9 cm, BC = 6 cm. Classify triangle and find all angles.

Solution:

- **Isosceles triangle.**
- **Use properties for angle calculation and well-labeled diagrams.**

Q: A triangle has sides 4 cm, 8 cm, 3 cm—is it possible?

- **$4 + 3 = 7 < 8 \Rightarrow$ Not possible (fails triangle inequality).**

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12. Final Premium Checklist

- **Diagrams: mark sides and angles, use tick marks/arcs.**
- **Always specify correspondence in congruence proofs.**
- **Each theorem: full statement, stepwise proof.**
- **Summarize error bank and self-check for valid reasoning.**

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