CHAPTER 6: TRIANGLES

Introduction

Triangles are one of the most important concepts in geometry and are widely used in mathematics as well as real-life applications such as construction, engineering, navigation, and design. In earlier classes, we studied the basic properties of triangles such as types, perimeter, and area.

In this chapter, we focus mainly on **similarity of triangles**, important theorems like the **Basic Proportionality Theorem**, and the famous **Pythagoras Theorem** along with its applications.

What is a Triangle?

A **triangle** is a polygon with three sides, three vertices, and three angles.

Basic Properties:

- Sum of interior angles = **180°**
- Sum of exterior angles = **360°**

Types of Triangles

Based on Sides:

1. **Scalene Triangle** – All sides are unequal
2. **Isosceles Triangle** – Two sides are equal
3. **Equilateral Triangle** – All sides are equal (each angle = 60°)

Based on Angles:

1. **Acute Triangle** – All angles < 90°
2. **Right Triangle** – One angle = 90°
3. **Obtuse Triangle** – One angle > 90°

Congruence vs Similarity

Basis	Congruence	Similarity
Shape	Same	Same
Size	Same	May differ
Sides	Equal	Proportional
Angles	Equal	Equal

Conclusion:
All congruent triangles are similar, but all similar triangles are not congruent.

Similarity of Polygons

Two polygons with the same number of sides are said to be **similar** if:

1. Their corresponding angles are equal
2. Their corresponding sides are proportional

Similarity of Triangles

Two triangles are similar if:

- Their **corresponding angles are equal**
- Their **corresponding sides are proportional**

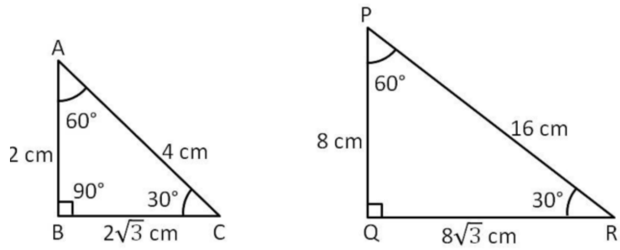
Notation:

ΔABC ~ ΔDEF

Criteria for Similarity of Triangles

1. AAA (Angle-Angle-Angle) / AA Similarity

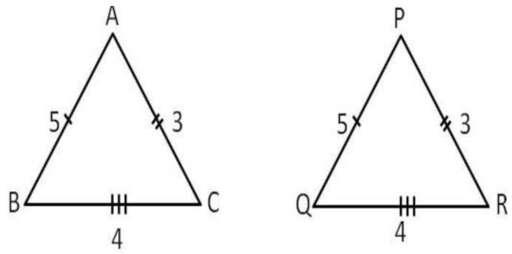
If **all angles** of one triangle are equal to corresponding angles of another triangle, then triangles are similar.



2. SSS (Si

If **corresponding sides** are proportional:

AB/DE = BC/EF =

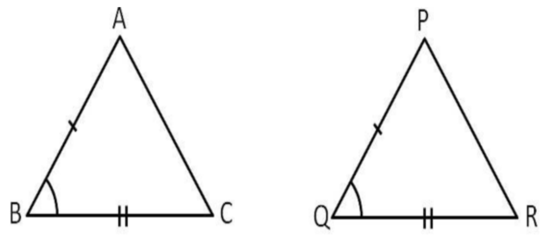


Then triangles are similar.

3. SAS (Side-Angle-Side) Similarity

If:

- One angle is equal
- The sides including that angle are proportional



Then triangles are similar.

Basic Proportionality Theorem (BPT)

Statement:

If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides those sides in the same ratio.

Formula:

AP / PB = AQ /

Converse of BPT:

If a line divides two sides of a triangle in the same ratio, then it is parallel to the third side.

Important Results of Similar Triangles

1. Ratio of Areas

If two triangles are similar:

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQ} = \frac{(AB)^2}{(PQ)^2}$$

2. Altitudes, Medians, and Perpendiculars

Corresponding altitudes and medians are in the same ratio as corresponding sides.

Right Triangle Properties

Perpendicular from Right Angle

If a perpendicular is drawn from the right angle to the hypotenuse:

- It divides the triangle into two smaller triangles
- All three triangles are similar

Pythagoras Theorem

Statement:

In a right-angled triangle:

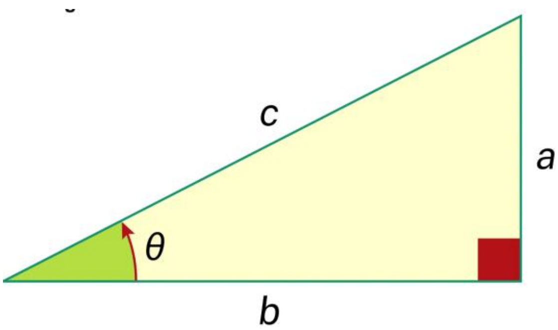
$$c^2 = a^2 + b^2$$

$$a$$

$$b$$

$$c = \sqrt{a^2 + b^2} \approx 21.21$$

$$a^2 + b^2 = c^2 \approx 225.00 + 225.00 = 450.00$$



Where:

- c = Hypotenuse
- a, b = Other two sides

Converse of Pythagoras Theorem

If in a triangle:

$c^2 = a^2 + b^2$

Then the triangle is a **right triangle**.

Applications of Pythagoras Theorem

- Finding unknown sides
- Checking right triangles
- Real-life applications (ladders, towers, distance)

Important Solved Concepts

Example 1: Check Right Triangle

Sides = 5 cm, 12 cm, 13 cm

Check:
$5^2 + 12^2 = 25 + 144 = 169 = 13^2$

✓ Triangle is right-angled

Example 2: Using BPT

If DE || BC, then:

AD/DB = AE/EC

Example 3: Area Ratio

If sides are in ratio 2:3

Then areas are in ratio:
$(2^2):(3^2) = 4:9$

Short Answer Concepts

- Equal angles → triangles similar
- Proportional sides → triangles similar
- Equal sides opposite equal angles
- Use BPT for parallel lines

Important Reasoning Points

- Correspondence order must be correct
- Ratios must match in correct sequence
- Parallel lines → proportional segments

- Area ratio uses square of sides

Frequently Asked Questions

Q1: Are two triangles similar if only one angle is equal?

❌ No, at least two conditions required

Q2: Can two triangles be similar but not congruent?

✓ Yes, if size differs

Q3: Does equal area mean similar triangles?

❌ No

Real-Life Applications

- Shadow problems (height & distance)
- Construction and architecture
- Navigation and mapping
- Physics and engineering

Practice Questions

MCQs (Sample)

1. If triangles are similar, their sides are:
   (a) equal
   (b) proportional ✓
2. Ratio of areas depends on:
   (a) sides
   (b) square of sides ✓

Short Questions

1. Check if triangle is right-angled
2. Use BPT to prove parallel lines
3. Find missing side using similarity

Long Questions

1. Prove Basic Proportionality Theorem
2. Prove Pythagoras Theorem
3. Solve height and distance problems

Key Formulas Summary

- AP/PB = AQ/QC
- Area ratio = $(\text{side ratio})^2$
- $c^2 = a^2 + b^2$

Learning Outcomes

After studying this chapter, students will be able to:

- Understand similarity of triangles
- Apply AAA, SSS, SAS criteria
- Use BPT in proofs
- Solve problems using Pythagoras theorem
- Apply concepts in real-life situations